熱傳遞 Heat Transfer

a discipline of thermal engineering that concerns the generation, use, conversion, and exchange of thermal energy (heat) between physical systems.

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Introduction

- 熱力學：based on the concept of equilibrium or infinitesimal deviations from equilibrium.
- 熱傳學：arises only from non-equilibrium, specifically, from finite differences of temperature.
- 熱傳、質傳、動量傳遞 (heat, mass, momentum transfer)、電流輸送等均為輸送現象 (transport phenomena)，具有共通的計算公式。
- Heat Transfer may be
  - Steady-state (穩態)：Temperature and heat fluxes do not change as functions of time.
  - Steady-periodic-state (階段重現性穩態)：conditions change with time in a regular fashion and periodically return to their starting conditions.
  - Transient-state (暫態)：不屬以上二者的狀態。
- For most engineering designs of agricultural buildings, steady-state analysis (穩態分析) is adequate at least as a close approximation.

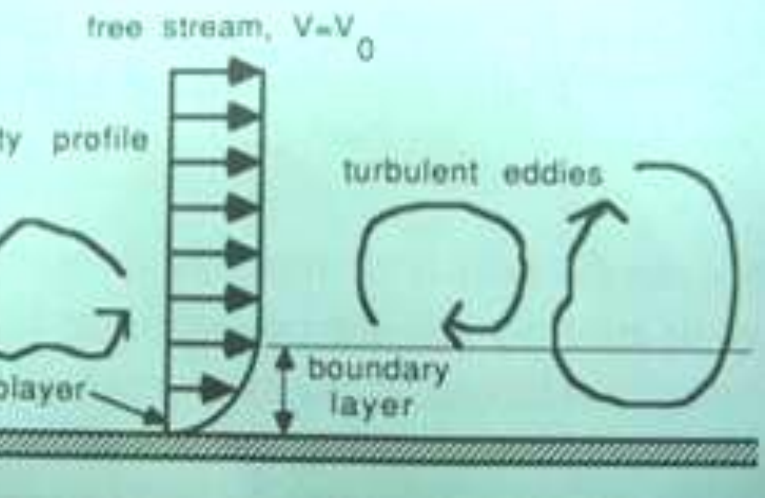
熱傳導 (Thermal Conduction)

- Thermal Conduction is the elastic collision in a fluid or the oscillations of atoms and transport of free electrons in a solid, continuous medium.
- 熱傳導是唯一可透過不透明固體的傳熱方式，其亦可發生於流體中，但僅限於平流 (laminar flow) 或擾流邊界層 (turbulent boundary layer)中極靠近物體表面的平流次層 (laminar sublayer)中。

熱對流 (Thermal Convection)

- 與流體相關，涉及流體內由一處傳至另一處的能量傳遞以熱對流(Thermal Convection)稱之，另有涉及流體與固體表面的能量傳遞以對流熱傳遞(Convective heat transfer)稱之。
  - 熱對流(Thermal Convection)主要係透過渦流運動 (Eddying motion)
  - 對流熱傳遞(Convective heat transfer)主要在邊界層(boundary layer)中涉及渦流運動與熱傳導。
- 渦流運動主要發生於擾流/紊流層與局部的邊界層 (又稱緩衝層 ，Buffer layer)，熱傳導則發生於平流次層(laminar sublayer)。

Figure 3-1

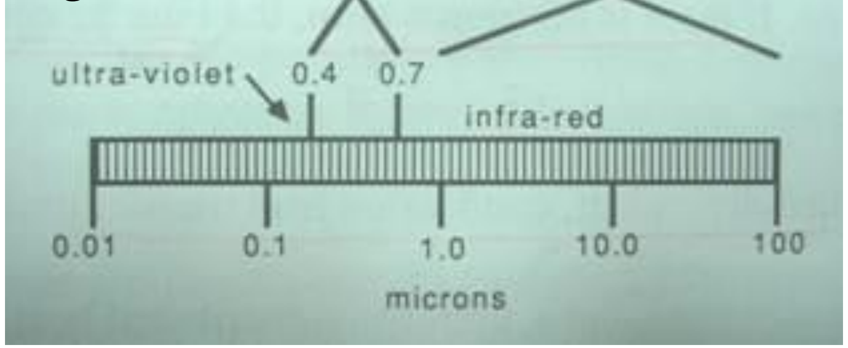


- 透過外力如風扇或幫浦去帶動流體者稱之為強制對流 (Forced convection)，透過由於溫差所造成的密度差而產生能量傳遞者稱之為自然對流 (Natural convection) 或自由對流 (Free convection)。

熱輻射 (Thermal Radiation)

- All objects at temperatures above absolute zero emit thermal radiation.
- 以往總認為只有紅外線 (infrared) 與熱輻射有關，事實上可見光或更短的紫外線照射到某物體之後，只要有被吸收，其亦會轉化為熱能。
- 紅外線的波段起至紅光之外 (700 nm, 0.7 micron or μm)，超過10 micron，甚至100 micron上限則無明確的定義。
- 常溫範圍的物體所發出之熱輻射之波段範圍的波峰落在10 micron附近，此部份的輻射線一般以遠紅外線稱之。See Figure 3-5.
- Wien’s law：The wavelength for peak emission intensity is found using Wien’s law，λ_{max} = 2897/T, where λ_{max} is wavelength in microns and T is the surface temperature, K, of the emitting object.

Figure 3-2



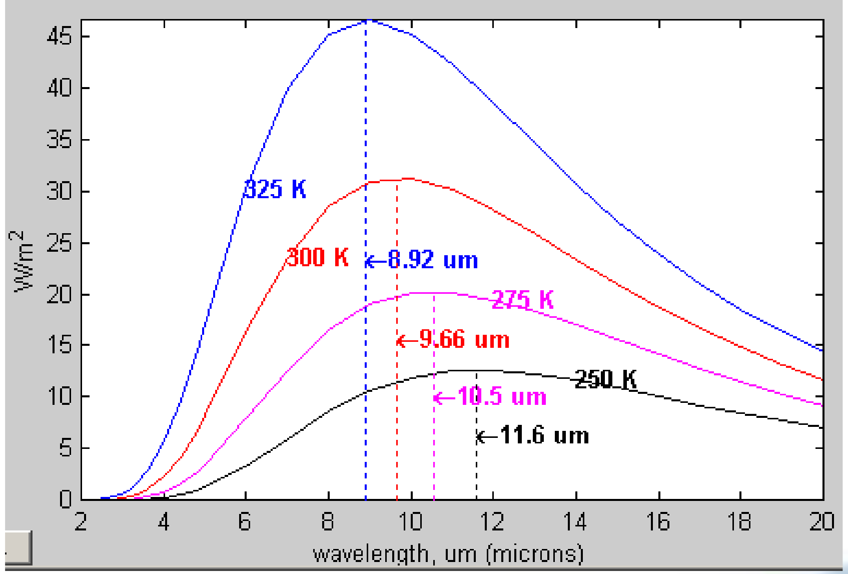


Figure 3-5

熱傳導

公式 $$\nabla^2 t + \frac{q_{gen}}{k} = (\alpha)^{-1} \bullet \frac{\delta t}{\delta \tau}$$

For isotropic solids：

t	Temperature	℃, K
q_{gen}	rate of internal heat generation	W/m^3
k	thermal conductivity	W/mK
α	thermal diffusivity	m^2/s
τ	time	s

- Thermal conductivity (熱傳導係數，k): intensive property,

  k = -q”/(dt/dn)

- 常見材料的熱傳導係數列於附錄A3-1, A3-2。(p.385)

三相材料的熱傳導係數 (W/mK) 範圍

氣體	0.008 – 0.6	空氣 0.0257 (at 20℃)
液體	0.09 – 0.7	水 0.594 (at 20 ℃)
固體	0.3 – 419	銀 419, 銅 386, 金 311 (at 20℃)

穿毛衣保溫
一旦毛衣濕了，失溫快

不同溫度範圍內的空氣熱傳導係數

空氣可有良好的隔熱保溫效果

溫度℃	熱傳導係數 W/mK	溫度℃	熱傳導係數 W/mK
-55	0.0200	60	0.0287
-20	0.0224	80	0.0302
0	0.0241	100	0.0316
20	0.0257	500	0.0562
40	0.0272	1000	0.0802

熱擴散

- Thermal diffusivity (熱擴散係數 or 熱擴散率，α):
  $\alpha = k/(\rho C_p)$, 傳熱能力 vs. 蓄熱能力
  a measure of how rapidly thermal energy can penetrate a solid material.
  - α的單位：(W/mK) / ((kg/m³)(W s/kgK)) = m^2/s
- 熱擴散係數在**time-dependent** 熱傳導問題中扮演重要角色

常見材料的熱擴散係數 (m^2/s)範圍

銀	$1.70*10^{-4}$	CO_2	$1.08*10^{-5}$
金	$1.18*10^{-4}$	水銀	$3.33*10^{-6}$
銅	$1.00*10^{-4}$	冰	$1.16*10^{-6}$
蒸氣	$2.28*10^{-5}$	雲母	$1.94*10^{-7}$
空氣	$2.17*10^{-5}$	水	$1.47*10^{-7}$

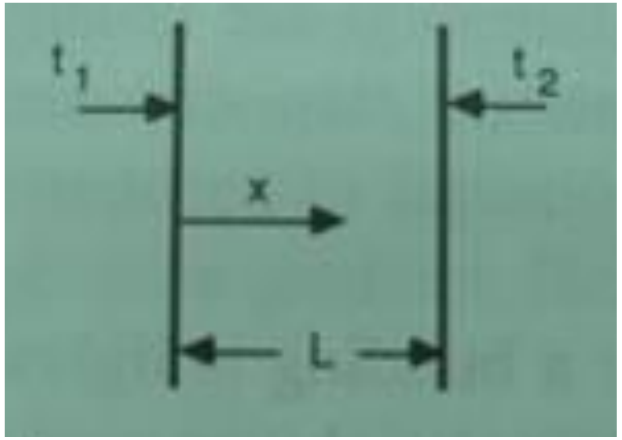
熱傳導公式的三個簡化型態

$$\nabla^2 t + \frac{q_{gen}}{k} = (\alpha)^{-1} \bullet \frac{\delta t}{\delta \tau}$$

Conditions	Name	Equation
no heat source	**the Fourier equation**	$\nabla^2 t = (\alpha)^{-1} \delta t / \delta \tau$
steady-state with a uniformly distributed heat source	**the Poisson equation**	$\nabla^2 t + q_{gen} / k = 0$
steady-state and no heat source	**the Laplace equation**	$\nabla^2 t = 0$

……..(3-7)

Conduction heat transfer: Laplace equation



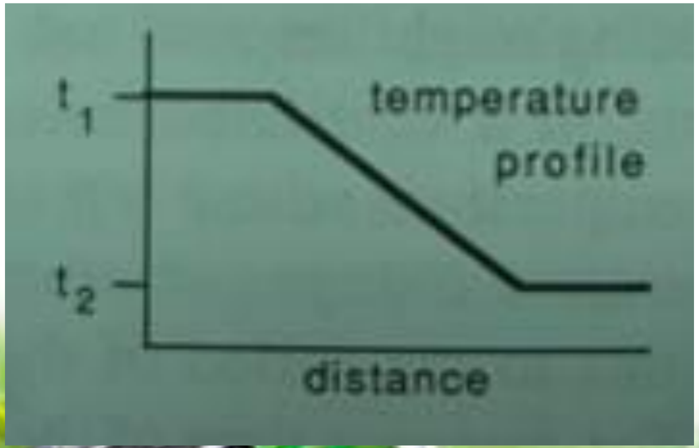
Ex. 3-1. 某厚度為 L 的均質牆壁兩側溫度為 t1, t2，求牆壁內的溫度梯度？

Eq 3-7 Laplace equation，直角坐標

$d^2 t / d x^2 = 0$ -- (3-8)

$t = C_1 x + C_2$ --- (3-9)

$t = t_1 + (t_2 - t_1)(x / L)$ --(3-10)



Ex. 3-2. 某外圈半徑為 r_o，內圈半徑為 r_i的環形圓柱，當內、外圈表面溫度分別為 ti 與 to，求圓柱截面的溫度場分佈。

Solution: Equation 3-7 applies again in the form

eq 3-4,

$$\frac{d^2t}{dr^2}+\frac{1}{r}\frac{dt}{dr}=0, \qquad (3\text{-}11)$$

which can be solved readily by substituting S = dt/dr. This reduces Equation 3-11 to

$$\frac{dS}{dr}+\frac{S}{r}=0. \qquad (3\text{-}12)$$

If Equation 3-12 is multiplied by rdr, it becomes

$$r\,dS+S\,dr=0, \qquad (3\text{-}13)$$

which is, by definition,

$$d(rS)=0. \qquad (3\text{-}14)$$

Equation 3-14 can be integrated once to

對 r 積分.

$$rS=c_1 \qquad (3\text{-}15)$$

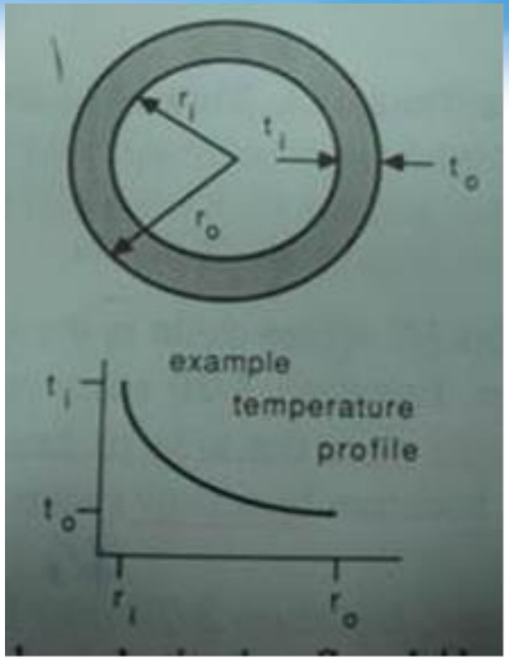
and a second time to

$$t=c_1\ln r+c_2. \qquad (3\text{-}16)$$

Applying the boundary conditions leads to

$$t_i=c_1\ln r_i+c_2, \text{ and} \qquad (3\text{-}17)$$

$$t_o=c_1\ln r_o+c_2. \qquad (3\text{-}18)$$



The constants of integration are determined using Equations 3-17 and 3-18, and the temperature field can be expressed as

$$t=\underbrace{\frac{t_o-t_i}{\ln(r_o/r_i)}}_{c_1}\ln r+\underbrace{\frac{t_i\ln r_o-t_o\ln r_i}{\ln(r_o/r_i)}}_{c_2} \qquad (3\text{-}19)$$

Equation 3-19 shows temperature in this steady state case is not linear in distance, but rather shows a logarithmic increase (or decrease, depending on the boundary conditions). Linearity would not be expected intuitively, of course, because the cross-sectional area of heat transfer is a function of radius, thus the temperature gradient must also be a function of radius for steady state heat transfer to occur.

Conduction heat transfer: Fourier equation

Thermal flux due to conduction heat transfer: $q'' = -k\, dt / dn$, (3-20)

Heat flux (q”): W/m^2, J/m^2.s
Heat flow (q): W, J/s

Conductivity (k): W/m.K
Conductance (U): W/m^2 .K

Resistance (R) = 1/U

in a restatement of Equation 3-2 where dt/dn is the temperature gradient and q” is heat flux by conduction. Equation 3-20 expresses the Fourier law of heat conduction. When integrated over the area of heat flow (area normal to the direction of flux) heat flux becomes heat flow (watts, for example).

Under steady-state conditions, Equation 3-20 may be integrated along the path and over the area of heat flow to yield heat flow, q

$$q = kA\Delta t / L; \Delta t = t_1 - t_2 \quad (3\text{-}21)$$

Equation 3-21 is frequently rearranged and the terms k/L grouped into a single term, U, the unit area thermal conductance,

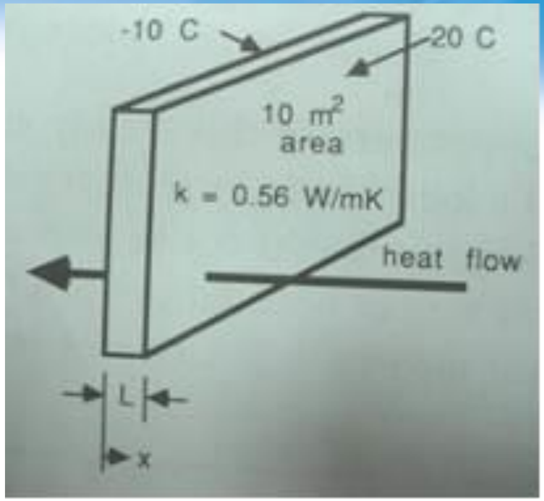
$$q = UA\Delta t; U = k / L. \quad (3\text{-}22)$$

Thermal conductance is an extensive property, whereas, thermal conductivity is intensive.

A final rearrangement is frequently made for convenience. The inverse of unit area thermal conductance, termed unit area thermal resistance, R, is used in a restatement of Equation 3-22,

$$q = A\Delta t / R; R = L/k. = \frac{1}{U} \quad (3\text{-}23)$$

Ex. 3-3. 某 20 cm 厚的水泥牆，截面積為 10 平方米，兩側的表面溫度分別為20 度C與 -10 度C，已知水泥的熱傳導係數為 0.56 W/mK，請計算沿厚度方向的溫度場分佈與熱傳導速率。



Solution: The temperature field is independent of thermal properties for one-dimensional, steady-state heat transfer by conduction, and properties are uniform and isotropic. Thus Equation 3-10 applies, where $t_1 = -10$ C, $t_2 = 20$ C and $x = 0$ where $t = -10$ C.

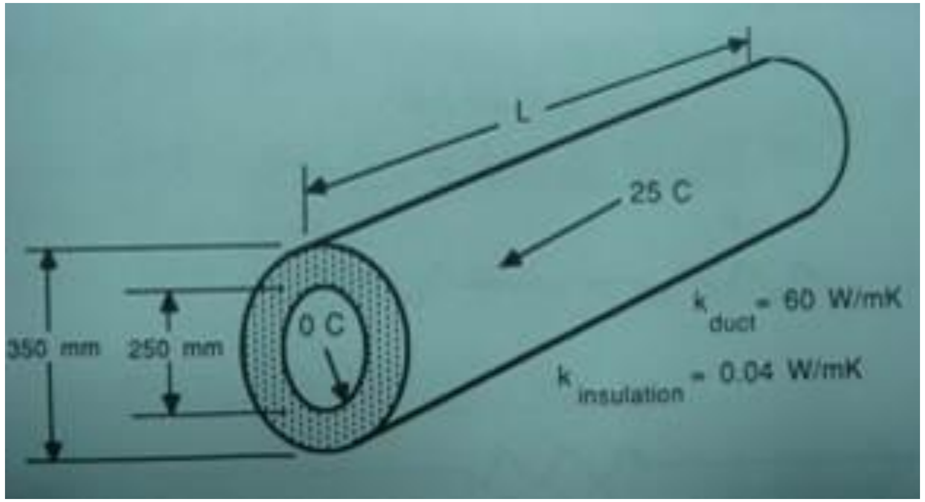
$$t = -10 + (20 - (-10))(x/0.2)$$
$$= -10 + 150 \quad (x \text{ in mm})$$

$$t = t_1 + (t_2 - t_1) \cdot (x/L)$$

and by Equation 3-23,

$$R = L/k = 0.2/0.56$$
$$= 0.357 \ m^2K/W$$
$$q = A\Delta t / R = (10)(30)/0.357$$
$$= 840 \text{ W.}$$

Ex. 3-4. 某蘋果冷藏庫使用圓柱狀金屬管將冷氣導入，金屬管厚度 1 mm，外徑 250 mm，外層包覆 50 mm 厚的保溫材料。金屬管內壁溫度為 0 ºC，外側絕緣材料的表面溫度為 25 ºC。金屬材質與絕緣保溫材料的熱傳導係數分別為 60 與 0.04 W/mK. 假設系統為穩態，請計算金屬管每一米長度由外界吸熱的速率 (in W/m)。



Solution: In cylindrical coordinates, thermal resistance to conductive heat transfer is

$$R = \frac{\ln(r_o / r_i)}{2\pi kL} \qquad (3\text{-}24)$$

where L is the length of the cylinder, and r_o and r_i are the outer and inner radii, respectively. It can be a useful exercise to develop this expression using the Fourier law of heat conduction, the definition of heat transfer using the resistance analogy (Equation 3-23), and the equation for the temperature field in cylindrical coordinates (Equation 3-19).

Note that Equation 3-24 presents thermal resistance differently than is the typical case for cartesian coordinates. For the cylinder, resistance is total thermal resistance for the cylindrical wall, not unit area or unit length thermal resistance (unless L = 1). This is an important distinction which will be emphasized later.

This example is to calculate heat gain per meter of duct, thus, L is 1 m in Equation 3-24, k is 0.04 W/mK for the insulation, and r_o and r_i are 175 and 125 mm, respectively. For now, consider just the insulation layer; it is likely to be the limiting resistance along the path of heat transfer. The duct wall itself will have relatively little resistance.

The resistance of one meter length of insulation is

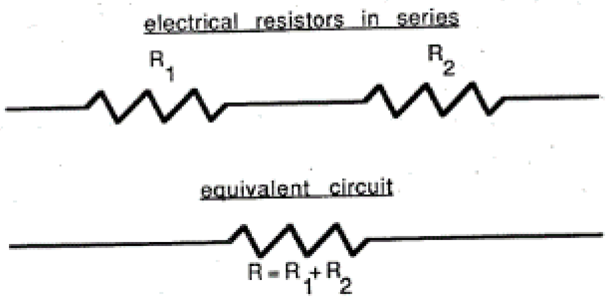
$$R = \ln(175/125)/(2\pi)(0.04)(1.0) = 1.34 \text{ mK/W}$$

and the heat gain per unit length is

$$q = \Delta t/R = (25 \text{ K})/(1.34 \text{ mK/W}) = 18.7 \text{ W/m}.$$

Resistances in Series

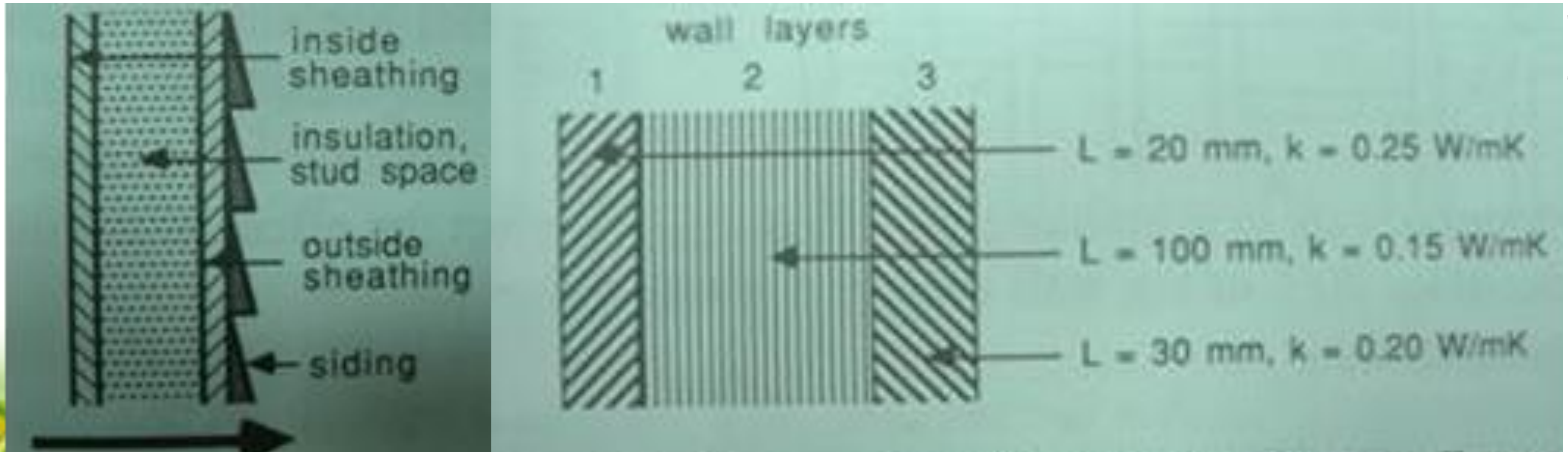
$$q = \Delta t / R'. \tag{3-25}$$



The electrical analogy is useful for realistic conduction heat transfer problems; standard electrical circuit equations can be used to simplify thermal circuits. In an electrical circuit with resistors in series, the total resistance of the circuit equals the sum of the individual resistances,

$$R' = \Sigma R_{individual}. \tag{3-26}$$

Ex. 3-5. 如圖 3-3 所示的牆壁，計算單位面積的熱阻與熱通量 (heat flux)，假設兩側溫度分別為 20 與 -5 度C。



igure 3-3. Cross-section of the wall considered in Examp

$R_1 = L_1/k_1 = (0.020\text{m})/(0.25\text{W/mK}) = 0.08\ \text{m}^2\text{K/W},$

$R_2 = L_2/k_2 = (0.100\text{m})/(0.15\text{W/mK}) = 0.67\ \text{m}^2\text{K/W},$

$R_3 = L_3/k_3 = (0.030\text{m})/(0.20\text{W/mK}) = 0.15\ \text{m}^2\text{K/W}.$

The total unit area thermal resistance is

$R' = 0.08 + 0.67 + 0.15 = 0.90\ \text{m}^2\text{K/W}.$

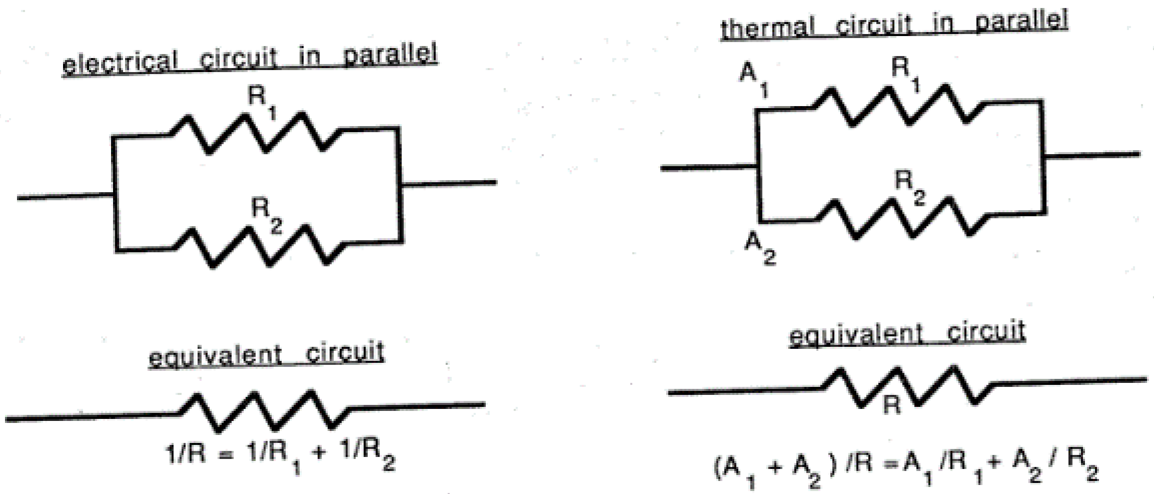
The heat flux is

$q' = \Delta t / R' = (20\ \text{C} - (-5\ \text{C}))/0.90\ \text{m}^2\text{K/W} = 27.8\ \text{W/m}^2.$

Resistances in Parallel

total resistance, R´, is found by the inverse rule,

$$1/R' = \Sigma\,(1/R_{individual}). \qquad (3\text{-}27)$$



$$A_{total}/R' = \Sigma\,(A_{individual}/R_{individual}) \qquad (3\text{-}28)$$

where A_{total} is the sum of areas of the heat loss paths,

$$A_{total} = \Sigma\,A_{individual}, \qquad (3\text{-}28a)$$

and $R_{individual}$ is the unit area thermal resistance of each path. In Equation 3-28, R´ is the unit area thermal resistance averaged over all heat transfer paths. Example 3-6 demonstrates an application of parallel heat transfer calculations.

Ex. 3-6

Problem: A wood-framed wall in a building (see Figure 3-4) is well insulated. However, framing occupies 20% of the wall area, and framing does not have the insulative effect of insulation.

Heat loss through such a wall is a situation of conductive heat transfer paths in parallel. One path of heat loss is through the framed part of the wall, the other path is through the insulated part. For this example, the framed part of the wall has a unit area thermal resistance of 2.3 m^2K/W, and the insulated part has a unit area thermal resistance of 4.1 m^2K/W.

If the wall is 3 m high and 10 m long, what is the average unit area thermal resistance of the wall, and what will be the heat loss through the wall when it is 20 C indoors and - 5 C outdoors?

Solution: Equation 3-28 is used to calculate the average unit area thermal resistance. There are two paths of heat loss. The total area of heat loss is 30 m^2 (3 m x 10 m). Framing occupies 20% of the wall, a heat loss area of 6 m^2 (0.20% of 30 m^2). The insulated part of the wall is the rest, 24 m^2. Equation 3-28 becomes

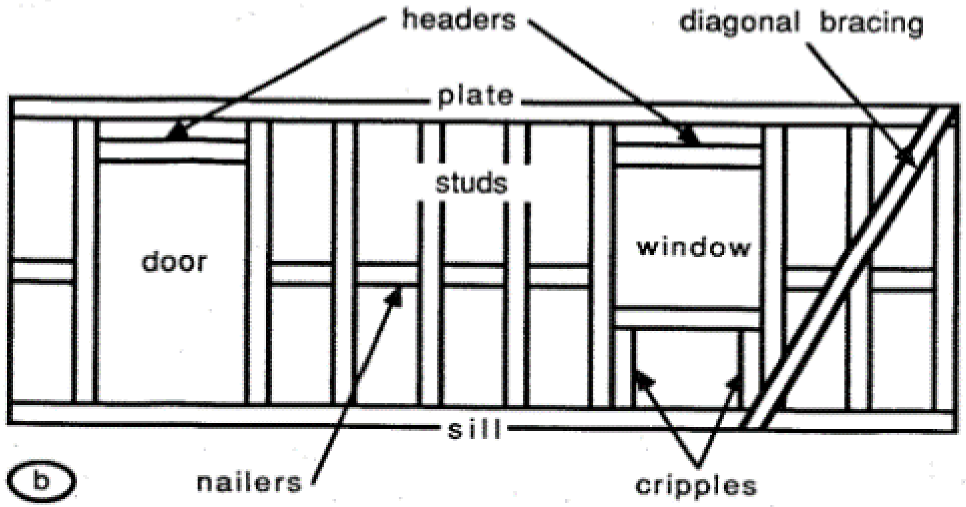
$$30\ m^2 / R' = (6\ m^2 / 2.3\ m^2K/W) + (24\ m^2 / 4.1\ m^2K/W).$$

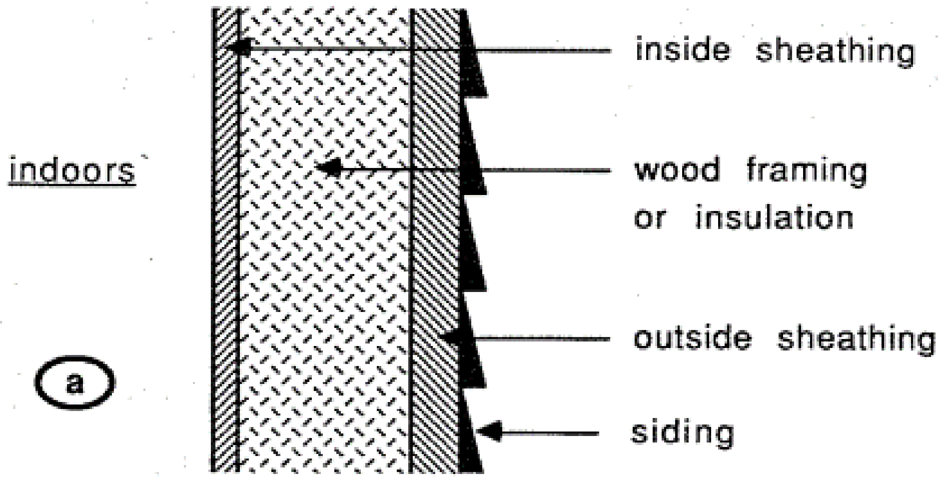
The average unit area thermal resistance is

$$R' = 3.55\ m^2K/W.$$

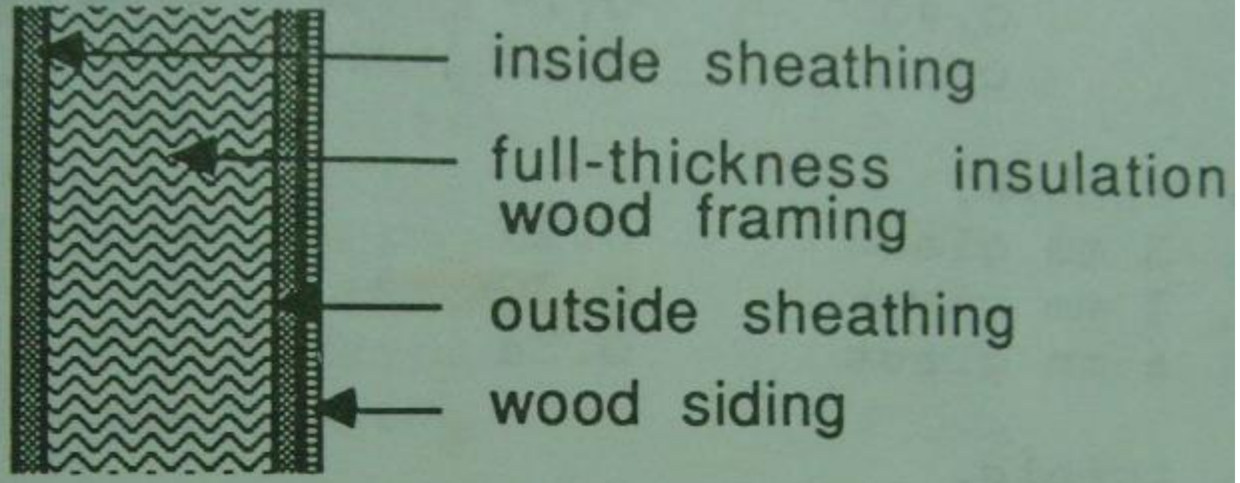
The heat loss through the wall can be calculated using Equation 3-23,

$$q = (30\ m^2)(20\ C - (-5\ C)) / 3.55\ m^2K/W, = 212\ W.$$

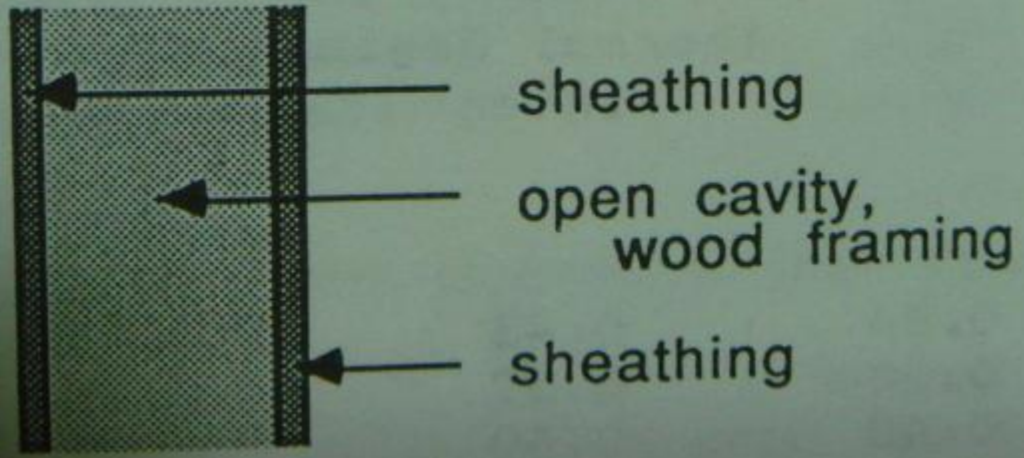




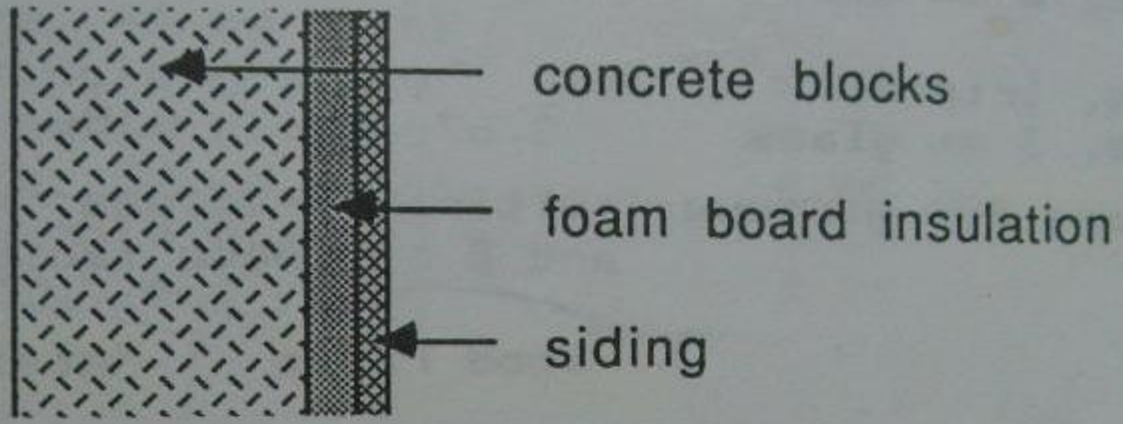
standard wood-framed outside wall



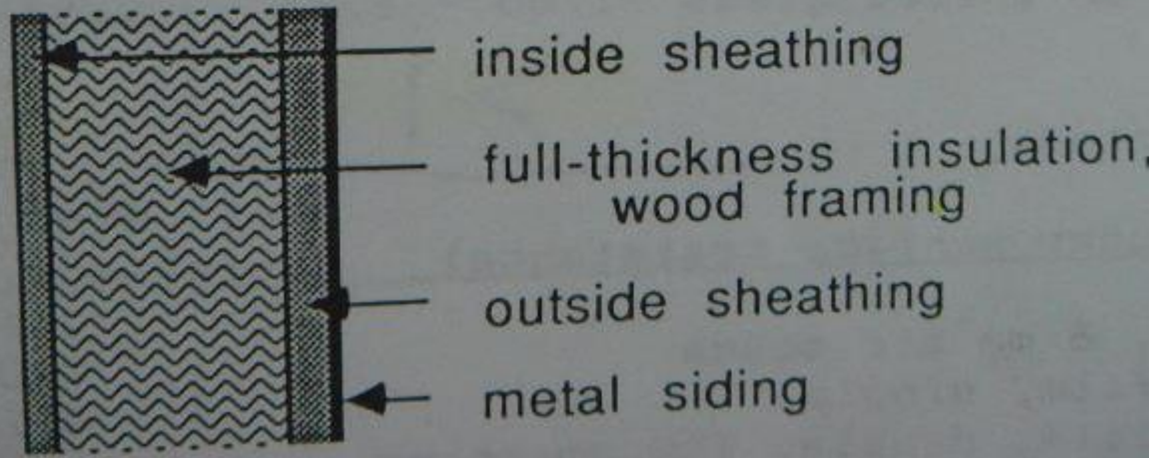
standard wood-framed partition wall



insulated concrete block wall



metal-sided wall with wood framing



對流熱傳遞 (Convective heat transfer)

自然對流 (free convection)

在探討自然對流的問題中，常見如右所示的三個無因次(dimensionless) 參數

Nusselt number	Nu = h L/k	L: a characteristic dimension of the solid object k: thermal conductivity of the fluid h: coefficient of the convective hea t transfer, fil coefficient, surface coefficient
	對流/傳導	The ratio of the ease with which heat is transferred by convection to the ease with which heat is transferred by conduction.
Prandtl number	Pr = μCp/k	μ: the dynamic viscosity of the fluid Cp: specific heat
	分子擴散/熱擴散	The ratio of the diffusion of momentum to the diffusion of heat from a solid surface through a boundary layer into a surrounding fluid.
	動黏度/熱擴散率	Pr = (μ/ρ)/α = (μ/ρ)/(k/(ρCp)) = μCp/k
Grashof number	$Gr=g\rho^2\beta L^3\Delta t/\mu^2$	β: the coefficient of thermal expansion g: gravitational constant
	浮力/黏滯力	The ratio of buoyancy forces to viscous drag forces within the fluid.

簡化計算

In general, natural convective heat transfer processes have been found to follow the relationship shown below:

$$Nu = c(Gr\ Pr)^n$$

where, n=0.33 for laminar flow and n= 0.25 for turbulent flow.

計算步驟：

(1). 求Gr, Pr, 由二者之乘值判斷是否為平流，決定 n 值。

平流：Gr Pr between 10^4 to 10^8.

擾流：Gr Pr between 10^8 to 10^{12}.

(2) 求出 Nu, 再由 Nu 之定義反求h (對流熱傳遞係數)

(3) 最後由 h = q” /△t or h = (q/A)/ △t 反求出 q or q”.

進一步簡化

- 由於 環控上經常面對的流體是空氣且溫度範圍較窄，求 h 的計算公式可予以簡化。
- 常溫範圍的空氣，GrPr 的計算可用 $10^8 L^3 \triangle t$
- 換言之，$L^3 \triangle t > 1$ 即為擾流，< 1 為平流。
- 依此可在 Table 3-2 找到適當的公式來計算 h 值。

For dry air at 20 °C and under standard ATM

Table 3-2. Natural convention heat transfer coefficients.

垂直板

Vertical plates

laminar range	$h = 1.42(\Delta t/L)^{0.25}$	(3-35)
turbulent range	$h = 1.31(\Delta t)^{0.33}$	(3-36)

水平板

面向下遇熱/面向上遇冷

氣流逼近

Horizontal plates facing upward when cooled or downward when heated (always laminar convective heat transfer)

	$h = 0.59(\Delta t/L)^{0.25}$	(3-37)

水平板

面向上遇熱/面向下遇冷

氣流遠離

Horizontal plates facing upward when heated or downward when cooled

laminar range	$h = 1.32(\Delta t/L)^{0.25}$	(3-38)
turbulent range	$h = 1.52(\Delta t)^{0.33}$	(3-39)

水平管

Horizontal cylinders

laminar range	$h = 1.32(\Delta t/L)^{0.25}$	(3-40)
turbulent range	$h = 1.24(\Delta t)^{0.33}$	(3-41)

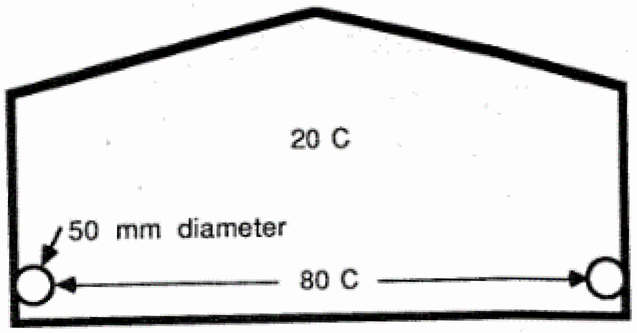
垂直管

Vertical cylinders can be treated as vertical plates.

(Δt in K, L in m, and h in W/m^2K)

Ex. 3-7.

Problem: Consider a horizontal heating pipe for a greenhouse. The pipe carri warm water and releases heat into the greenhouse air by convective he transfer. The outside diameter of the pipe is 50 mm, and its surface temperatu is 80 C. The greenhouse air temperature is 20 C. What will be the surfa convective heat transfer coefficient, and what will be the rate of convective he loss from the pipe?



Solution: The pipe can be considered a horizontal cylinder (no fins on the p were mentioned). However, to know which of the equations in Table 3-2 to u we must determine whether the convection will be laminar or turbulent. Fi Equation 3-34,

$$\begin{aligned} GrPr &= 10^8(0.050\ \text{m})^3(80\ \text{C} - 20\ \text{C}) \\ &= 0.75E+6 \end{aligned}$$

which is within the laminar range. The convective heat transfer coefficient thus be calculated as (using Equation 3-40)

$$\begin{aligned} h &= 1.32(60\ \text{K} / 0.050\ \text{m})^{0.25} \\ &= 7.8\ \text{W/m}^2\text{K}. \end{aligned}$$

The rate of heat loss can be determined by rearranging Equation 3-3 follows:

$$\begin{aligned} q'' &= h\Delta t \qquad (3 \\ &= (7.8\ \text{W/m}^2\text{K})(60\ \text{K}) = 470\ \text{W/m}^2. \end{aligned}$$

Heat loss from pipes is often expressed per unit length of pipe rather thar area. Each meter of pipe length corresponds to

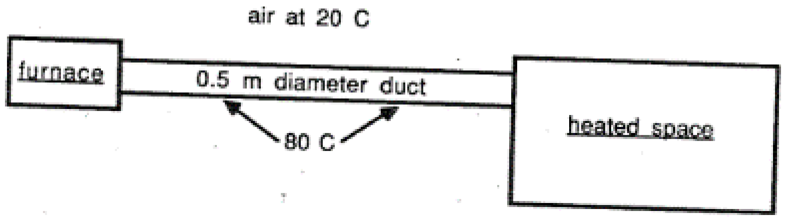
$$A = 2\pi rL = \pi(0.050\ \text{m})(1.0\ \text{m}) = 0.157\ \text{m}^2$$

thus,

$$q = (470\ \text{W/m}^2\ \text{K})(0.157\ \text{m}^2/\text{m}) = 74\ \text{W/m}.$$

Ex. 3-8.

<u>Problem:</u> Consider a horizontal sheet metal duct which carries heated air from a furnace to a heated space. The outside diameter of the duct is 0.5 m, and its outside surface temperature is 80 C. The air temperature in the space through which the duct passes is 20 C. What will be the surface convective heat transfer coefficient and the rate of heat loss by convection from the duct?



<u>Solution:</u> This example is similar to Example 3-7, only the dimensions have changed. Again, first check to determine whether laminar or turbulent conditions apply,

$$GrPr = 10^8(0.5\ m)^3(80\ C - 20\ C)$$
$$= 3.0E+9,$$

which is in the <u>turbulent range</u>. The convective heat transfer coefficient can, thus, be calculated by (using eq 3-41).

$$h = 1.24(60\ K)^{0.33}$$
$$= 4.79\ W/m^2K.$$

The rate of convective heat transfer can be calculated as in Equation 3-42,

$$q'' = (4.79\ W/m^2K)\ (60\ K)$$
$$= 287\ W/m^2,$$

, $q' = h \cdot \Delta t$

Each meter of duct has a surface area of

$$A = \pi(0.5\ m)(1.0\ m) = 1.57\ m^2,$$

thus, the heat loss by convection is

$$q = (287\ W/m^2)(1.57\ m^2/m) = 451\ W/m.$$

與前例比較，管的外徑(D)不同

但本例為紊流狀態，

h 較小 (4.79 < 7.8 W/m^2.K)

thermal flux q" 較小(287 < 470 W/m^2)

但 thermal flow q 較大，單位長度的流失的熱量增加很多 (451 > 74 W/m)

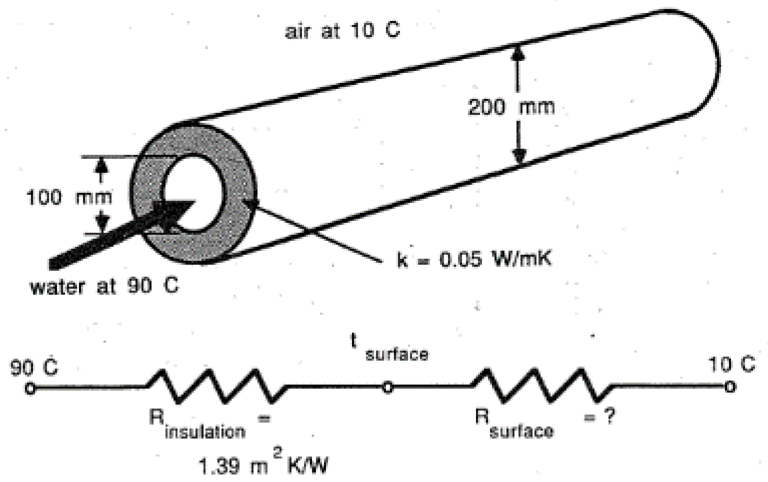
Ex. 3-9.

Problem: A hot water pipe is insulated to prevent heat loss from hot water as it flows between the water heater and the point of use.

The pipe's outside diameter is 100 mm, the insulation thickness is 50 mm, the pipe temperature is essentially that of the hot water, 90 C, and air temperature surrounding the pipe is 10 C.

The thermal conductivity of the insulation on the pipe is 0.05 W/mK.

Calculate the heat flux from the surface of the insulation, and the heat loss per meter of pipe.



This problem can be viewed as a thermal circuit with two resistances in series – the insulation, and the convective heat transfer resistance between the surface of the insulation and the air (a surface resistance). The resistance of the insulation can be calculated using Equation 3-24 P. 57

$$R_{insulation} = (\ln(0.10\text{ m} / 0.05\text{ m}))/(2\pi)(0.05\text{ W/mK})(1.0\text{ m}) = 2.21\text{ m K/W}.$$

Recall this thermal resistance is based on a unit length of pipe. Convective thermal resistance is based on unit area. We must decide which basis to use. Either will work, but for this example use unit area. The convective thermal resistance is based on the area of the outside of the insulation layer; the insulation resistance will be based on the same area.

Each meter length of pipe has a surface area of

$$A = \pi(0.20\text{ m})(1.0\text{ m}) = 0.628\text{ m}^2/\text{m}$$

Therefore, the unit area thermal resistance (based on the surface area) is

$$R_{insulation} = (2.21\text{ m K/W})(0.628\text{ m}^2/\text{m}) = 1.39\text{ m}^2\text{K/W}.$$

To begin the solution, assume conditions are laminar. An energy balance written for the outside surface of the insulation is

$$h_{surface}(t_{surface} - 10\text{ C}) = (90\text{ C} - t_{surface})/R_{insulation}$$

With the assumption of laminar airflow,

$$h_{surface} = 1.32((t_{surface} - 10\text{ C})/0.2\text{ m})^{0.25} = 1.97(t_{surface} - 10\text{ C})^{0.25}.$$

The energy balance can be rewritten as

$$1.97(t_{surface} - 10)^{1.25} = (90 - t_{surface})/1.39.$$

$$2.74(t_{surface} - 10)^{1.25} + t_{surface} = 90$$

$t_{surface}$	LHS
20 C	68.7
25	105.9
23	90.6
22.9	89.9
22.92	90.0

Considering significant digits, a surface temperature of 23 C is estimated. Now check the assumption of laminar flow.

$$GrPr = 10^8(0.2\ m)^3(23\ C - 10\ C)$$
$$= 1.1E+7$$

This is within the laminar range; the initial assumption was correct. Several procedures can now be used to calculate heat flux from the pipe. One way is to calculate the surface convective thermal resistance, the total thermal resistance, and the heat flux.

$$h_{surface} = 1.32((23\ C - 10\ C) / 0.2\ m)^{0.25}$$
$$= 3.75\ W/m^2K,$$

and $R_{surface} = 1/3.75\ W/m^2K = 0.267\ m^2K/W$.

The total series resistance is

$$R_{total} = 1.39\ m^2K/W + 0.267\ m^2K/W = 1.66\ m^2K/W,$$

and the heat flux is

$$q'' = (90\ C - 10\ C) / 1.66\ m^2K/W = 48\ W/m^2.$$

Each meter length of pipe has 0.628 m^2 surface area, thus, heat loss per meter is

$$q = (48\ W/m^2)(0.628\ m^2/m) = 30\ W/m.$$

強制對流 (forced convection)

在探討強制對流問題時，常見的兩個無因次 (dimensionless) 參數

Nusselt number	Nu = h L/k	L: a characteristic dimension of the solid object k: thermal conductivity of the fluid h: coefficient of the convective heat transfer, film coefficient, surface coefficient
Reynolds number	Re = ρV L/μ	μ: the dynamic viscosity of air ρ: mass density V: average velocity of fluid flow = Volume/A L: characteristic length
	分子力/黏滯力	The ratio of momentum forces to viscous forces and expre the level of turbulence. 代表著擾流擾動的程度

強制對流 (forced convection)

強制通風多屬擾流，輸送管中強制吹送的風，其 h 值可用下式求出：

$$h = c\ G^{0.8}/D^{0.2}$$

其中， c is related to thermal properties of air, 可查表3-3,
G = ρ*Volume，mass flow of air in the duct
D= 4 *(area)/(perimeter), 水力直徑

$= 4\pi r^2 / 2\pi r = 2r$ = 直徑 for 圓管

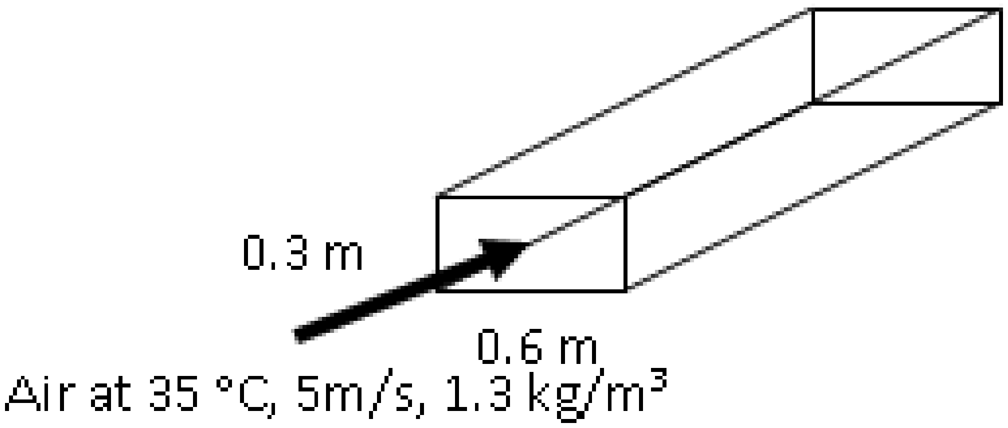
Table 3-3. Coefficient c in Equation 3-44 (SI units).

Temperature C	c
-18	3.09
4	3.18
27	3.21
49	3.26
71	3.32
93	3.37

Adapted from the *ASHRAE Handbook of Fundamentals* : for h in W/m^2K; G in kg/m^2s; D in m. An approximation of the data is the equation c = 3.14783 + 0.00240267t.

Ex. 3-10 (p.72)

35°C的空氣流過長方形截面 (0.3 m x 0.6 m) 的加熱管，空氣密度 1.3，風速 5 m/s，請求出管內的對流熱傳係數



$$h = c\, G^{0.8}/D^{0.2}$$

表 3-3，溫度35 °C，c= 3.23

mass flow rate G = 1.3 kg/m^3 * 5 m/s = 6.5 kg/(m^2s)

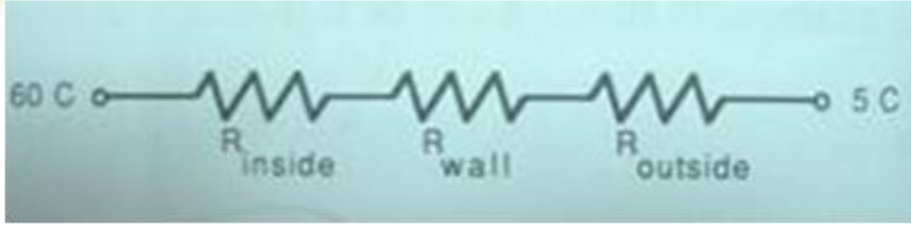
hydraulic diameter D = 4 * 0.18 m^2 / 1.8 m = 0.4 m

The convective heat transfer coefficient (h)

$= (3.23)\,(6.5)^{0.8}/(0.4)^{0.2}$

= 17.3 W/m^2K

Ex. 3-11 (p.73)



已知 $R_{outside} = 0.1$
R_{inside} 可以由 eq.44 計算，其中 c = 3.29 @ 60 °C (Table 3-3)

$$D = 0.8 \text{ m},$$
$$G = (0.9 \text{ kg/m}^3)(4 \text{ m}^3/\text{s})/(\pi)(0.4^2) = 7.16 \text{ kg/m}^2\text{s, and}$$

$$h = (3.29)(7.16 \text{ kg/m}^2\text{s})^{0.8}/(0.8 \text{ m})^{0.2} = 16.6 \text{ W/m}^2\text{K}.$$

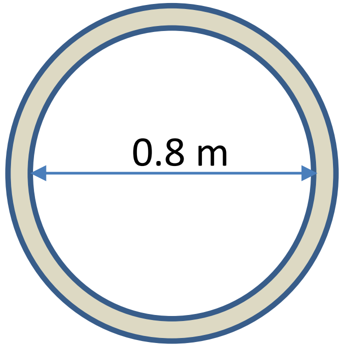
The unit area convective resistance is the inverse,

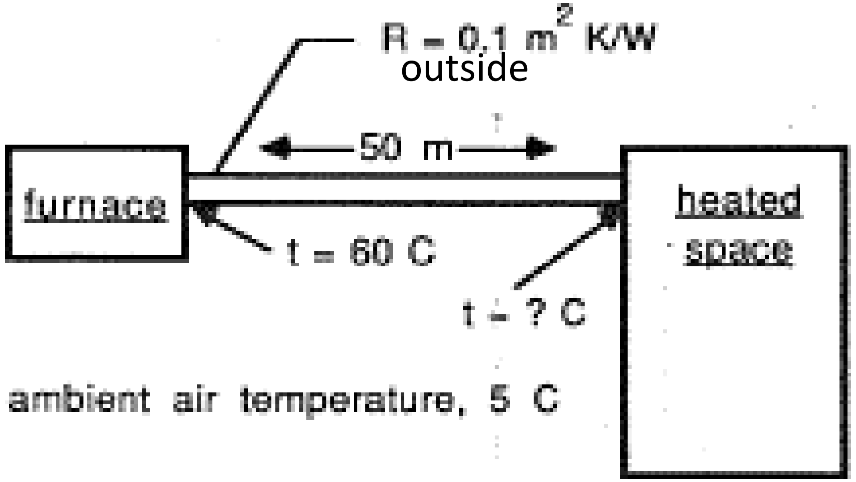
$$R_{inside} = 1 / 16.6 \text{ W/m}^2\text{K} = 0.060 \text{ m}^2\text{K/W}.$$

Problem: Air is heated in a furnace and distributed to a heated space through a round sheet metal duct at a volumetric flow rate of 4 m³/s. The duct diameter is 0.8 m and the outer surface of the duct is covered with 10 mm of expanded polyurethane having a thermal conductivity of 0.023 W/mK. See P387

The duct is 50 m long and passes through an unheated space where air temperature is 5 C. The surface resistance outside the duct insulation is 0.1 m^2K/W and includes both convective and radiation heat transfer. Density of the heated air is expected to be 0.9 kg/ m^3.

If air leaves the furnace at 60 C, what will be its temperature at the end of the 50 m long duct? At what rate will heat be lost from the heated air?





$$L = 1.0/(0.8\pi) = 0.398 \text{ m.}$$

Equation 3-24 is used to calculate the thermal resistance of the wall (insulation) for a duct length of 0.398 m (r_o= 0.41 m, r_i = 0.40 m)

$$R_{wall} = (\ln(0.41 / 0.40))/(2\pi)(0.023)(0.398)$$
$$= 0.429 \text{ m}^2\text{K/W.}$$

- 以上使用圓柱座標計算熱阻的公式
- 如果改以直角坐標的公式進行計算
  - $R = L/k = 0.01 \text{ m}/0.023 \text{ W/m.K} = 0.435 \text{ m}^2\text{.K/W}$
  - 非常接近圓柱座標的計算值
- 以外徑為基礎，R_{inside} 修正為 0.062
- $R_{total} = 0.062 + 0.429 + 0.1 = 0.591 \text{ m}^2\text{.K/W}$
  - 想想如果沒有保溫，$R_{total} = 0.062 + 0.1 = 0.162 \text{ m}^2\text{.K/W}$

- 接著計算管中熱空氣的溫度變化

Heat transfer through the wall of the elemental length can be written (Equation 3-23) as

$$q = A\Delta t / R;\ A = \pi D dL = 0.82\pi dL = 2.58 dL.$$

We have calculated the unit area thermal resistance, $R_{total} = 0.591\ m^2K/W$, thus,

$$q = 4.36(t_{air} - 5\ C)dL.$$

This thermal exchange must be balanced by heat loss from the mass of air flowing through the element, m,

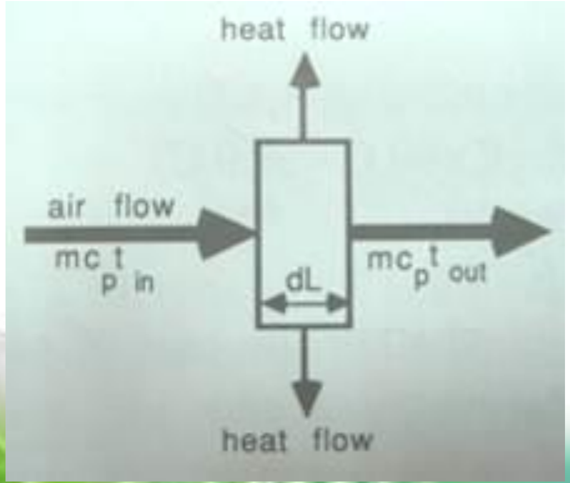
$$q = -mc_p dt_{air};\ m = (0.9\ kg/m^3)\ (4\ m^3/s) = 3.6\ kg/s.$$

The negative sign is introduced so a positive heat loss is associated with a negative temperature change.

If the specific heat of air within the duct is approximated as 1006 J/kgK, heat loss can be written as

$$q = -3620 dt_{air}.$$

Heat loss must equal heat gain, thus,



$$4.36\ (t_{air} - 5)\ dL = -3620 dt_{air}$$

which can be rearranged and integrated along the 50 m length of the duct in the form

$$\int_{60\ C}^{t_{exit}} \frac{dt_{air}}{(t_{air} - 5)} = -\int_{0\ m}^{50\ m} 0.12E-2\ dL$$

and solved as

$$t_{exit} = 5 + 55\ \exp\ (-0.056) = 57.0\ C.$$

The energy loss equation can be used again to estimate the rate of heat loss from the air where Δt is now a temperature change not a temperature difference.

$$\begin{aligned} q &= mc_p\Delta t \\ &= (3.6\ kg/s)(1006\ J/kgK)(60\ C - 57.0\ C) \\ &= 10{,}900\ W\ (\text{or } 10.9\ kW). \end{aligned}$$

(A natural next question is whether the cost of added insulation would be balanced by the value of heat energy saved.)

In general terms, air temperature in a process such as this example can be calculated from

$$t_{exit} = t_{ambient} + (t_{initial} - t_{ambient})\ \exp\ (-A/mc_pR)$$

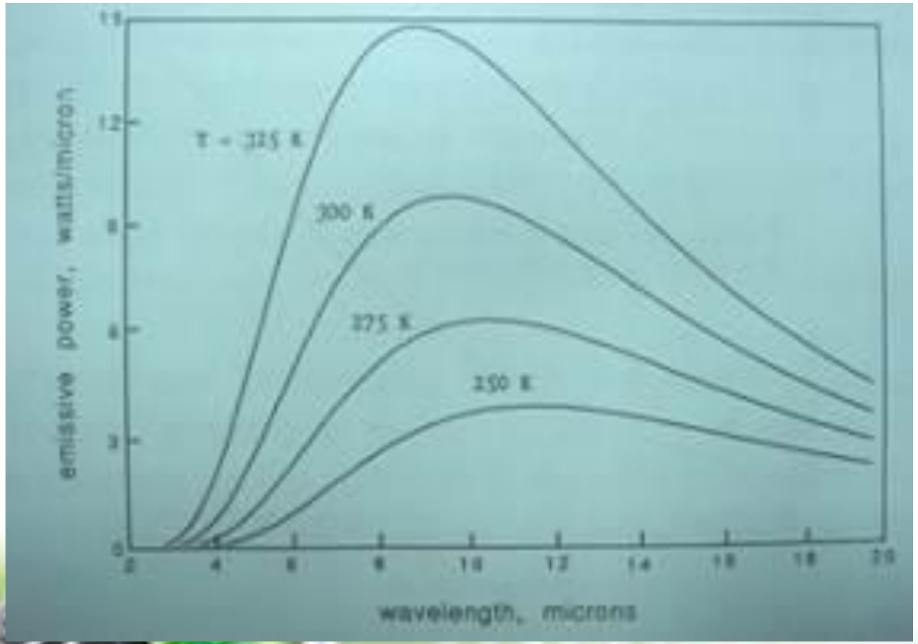
The simpler approach provides a check on the accuracy of the first approach. The unit area series thermal resistance is $0.591\ m^2K/W$, and the total area of heat transfer is $\pi(0.82\ m)(50\ m) = 129\ m^2$.

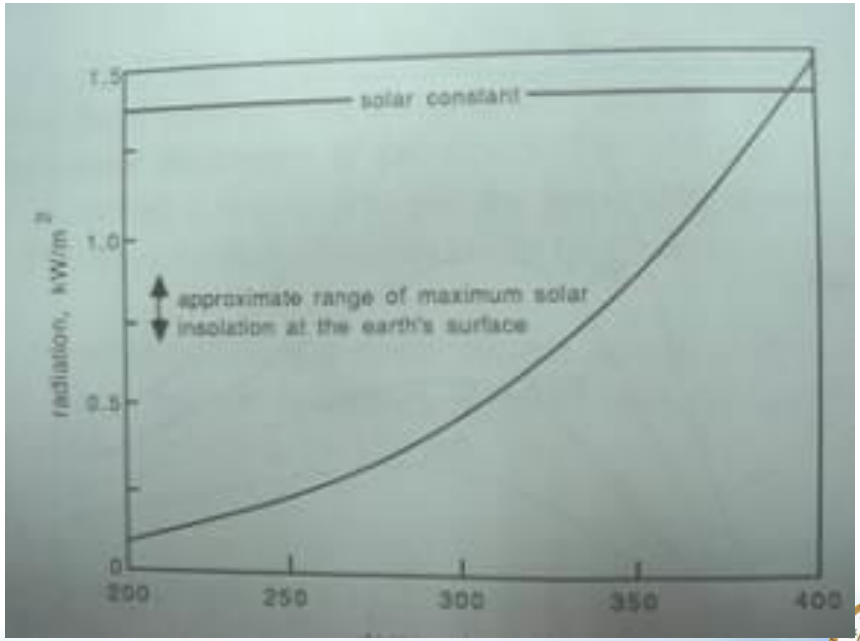
Thus

$$\begin{aligned} q &= A\Delta t / R = (129\ m^2)(55\ K) / 0.591\ m^2K/W \\ &= 12{,}000\ W \end{aligned}$$

輻射熱傳遞

- 馬克思 普郎克 (1858 ~ 1947) 德國物理學家
- 1900 發表一篇討論黑體輻射的論文，量子物理學創始人
- 1918 諾貝爾物理獎得主





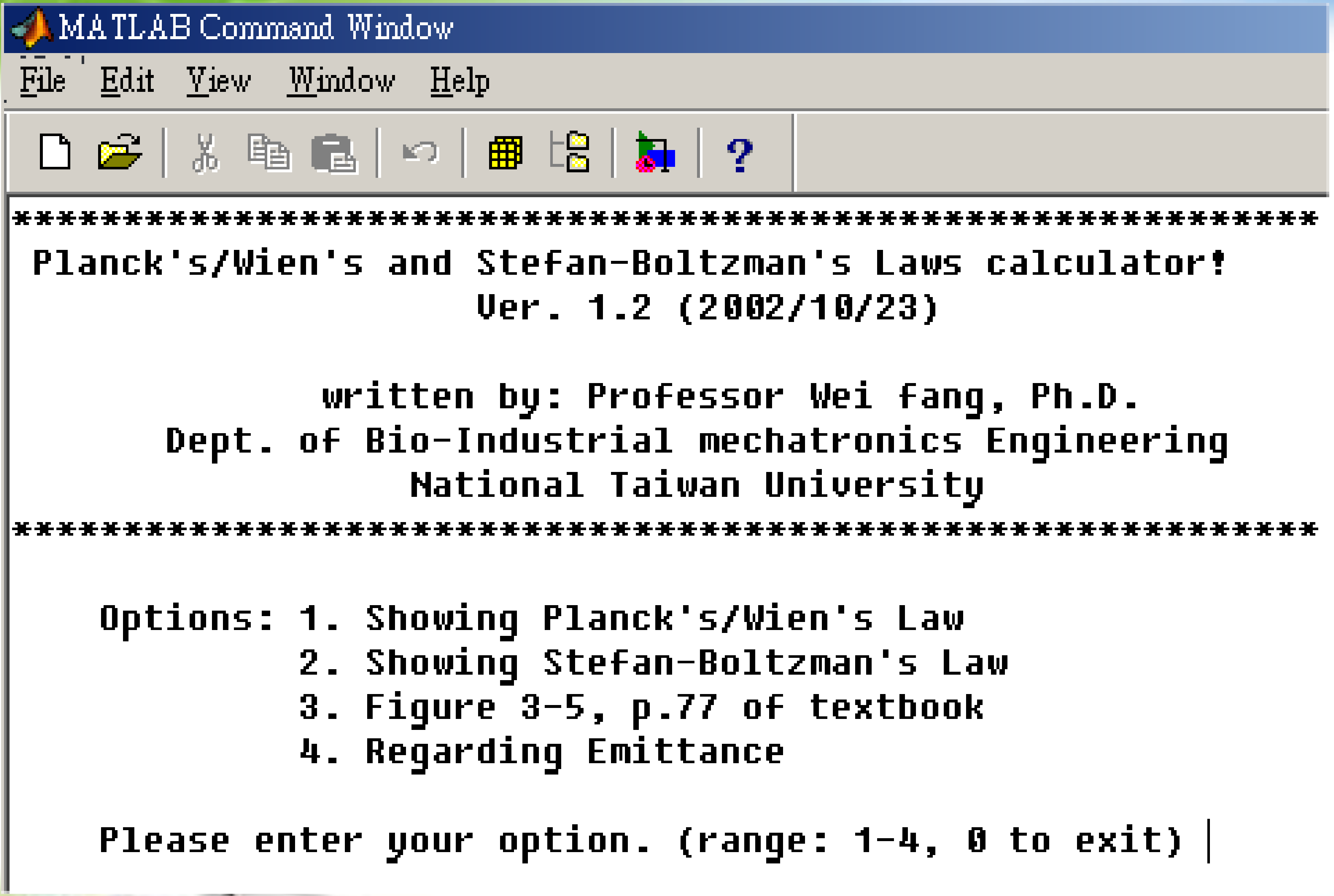


圖 1. 在Matlab Command Window 中輸入 plancks 的結果

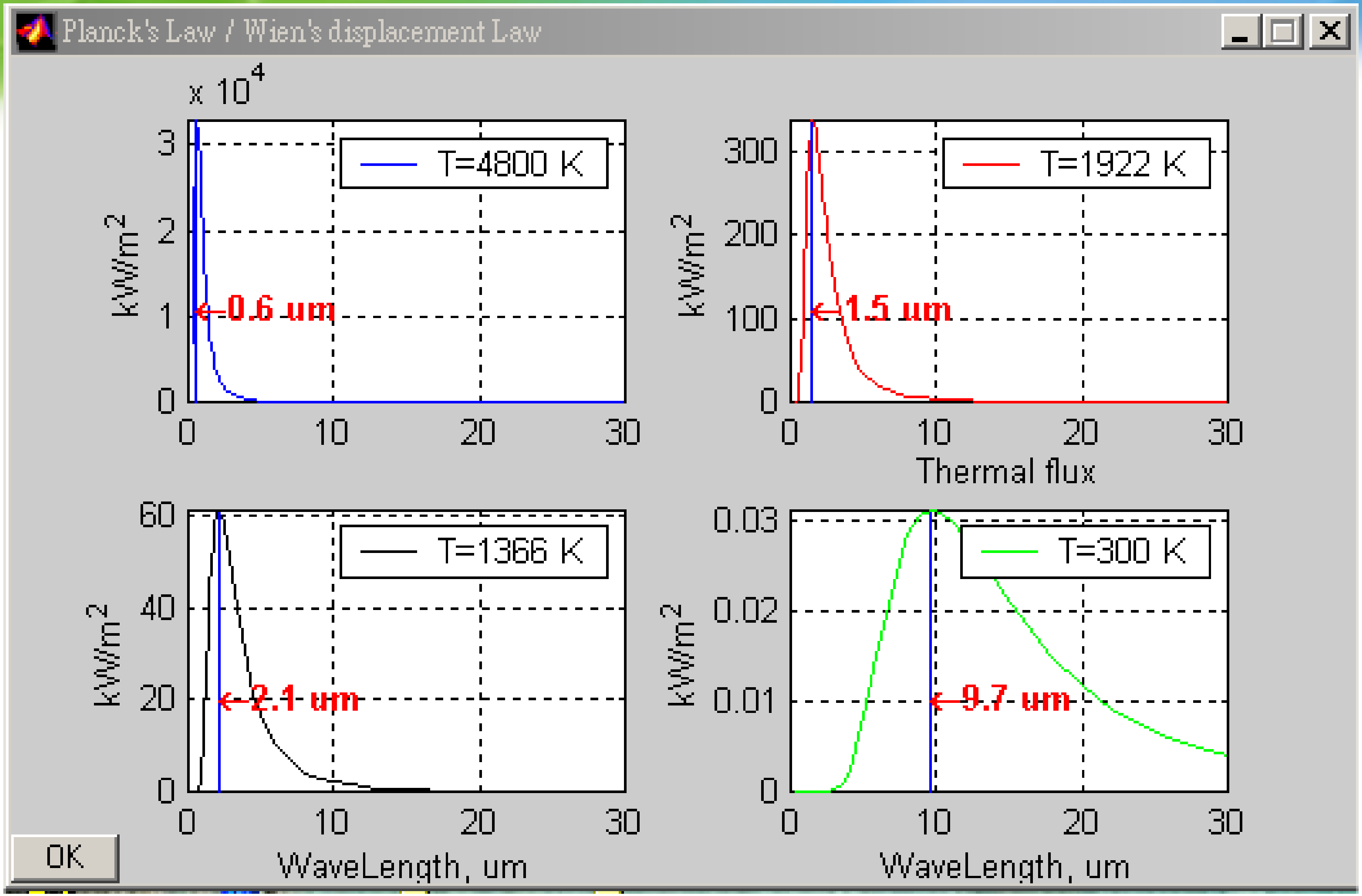


圖 2.　在 Matlab Command Window 中輸入 plancks(1) 的計算結果

圖 3-5 中各曲線的波峰的位置，遵循 Wien's Displacement law. 溫度愈高者，波峰的位置朝左移動(波長愈短)。

$$\lambda_{max}=2897/T \quad \text{(3-47)}$$

其中，λ_{max}單位為 micron, μm，T單位為絕對溫度 (K)

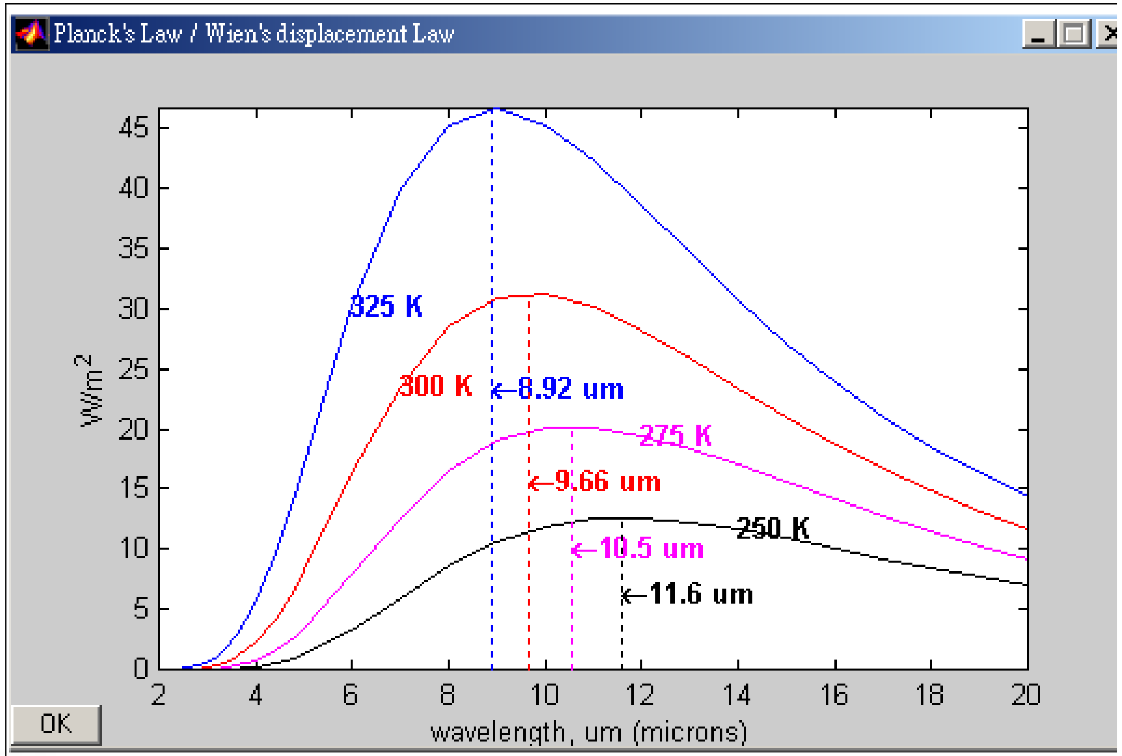


圖 3. 在 Matlab Command Window 中輸入 plancks(3) 的計算結果，此圖與教科書上 p.77 之圖 3-5 相同，唯，Y 軸範圍有差異，應是教科書上之圖錯誤。

圖 3-5之各曲線可用Planck's Law 描述，其計算公式如下：

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left(e^{C_2/\lambda T} - 1\right)}$$

其中	λ: 波長	(μm)
	$E_{b\lambda}$: 黑體的單色放射能	$(W / (\mu m)^2)$
	C1: 3.7413×10^8	$(W^*(\mu m)^4/m^2)$
	C2: 1.4388×10^4	(μm. K)

圖 3-5中曲線下面積之總和即為該溫度之物體所輻射出的通量(radiant flux)，可使用Stephan-Boltzmann relationship計算。

$q'' = \sigma T^4$ ………………… (3-48)

其中，σ 為Stephan-Boltzmann constant = 5.6697 E-8 W/m^2K^4.

上式在程式中常改寫為 $q'' = 5.6697 * (T/100)^4$

在大氣層外圍正向太陽的平面上(A surface normal to the sun)，所接受到的太陽能的值隨日期而異，其全年的平均值以太陽常數 (Solar Constant)稱之，其值為 1.353 kW/m^2.

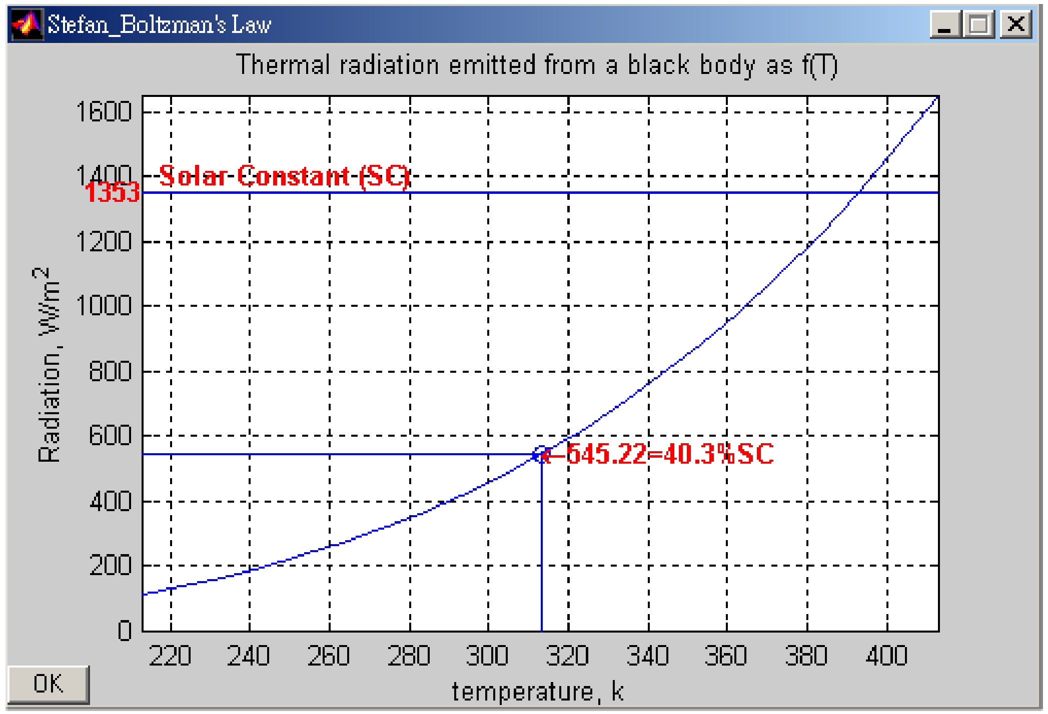


圖 4. 在 Matlab Command Window 中輸入 plancks(2) 的計算結果，此圖與教科書上之圖 3-6 相同。

Solar constant

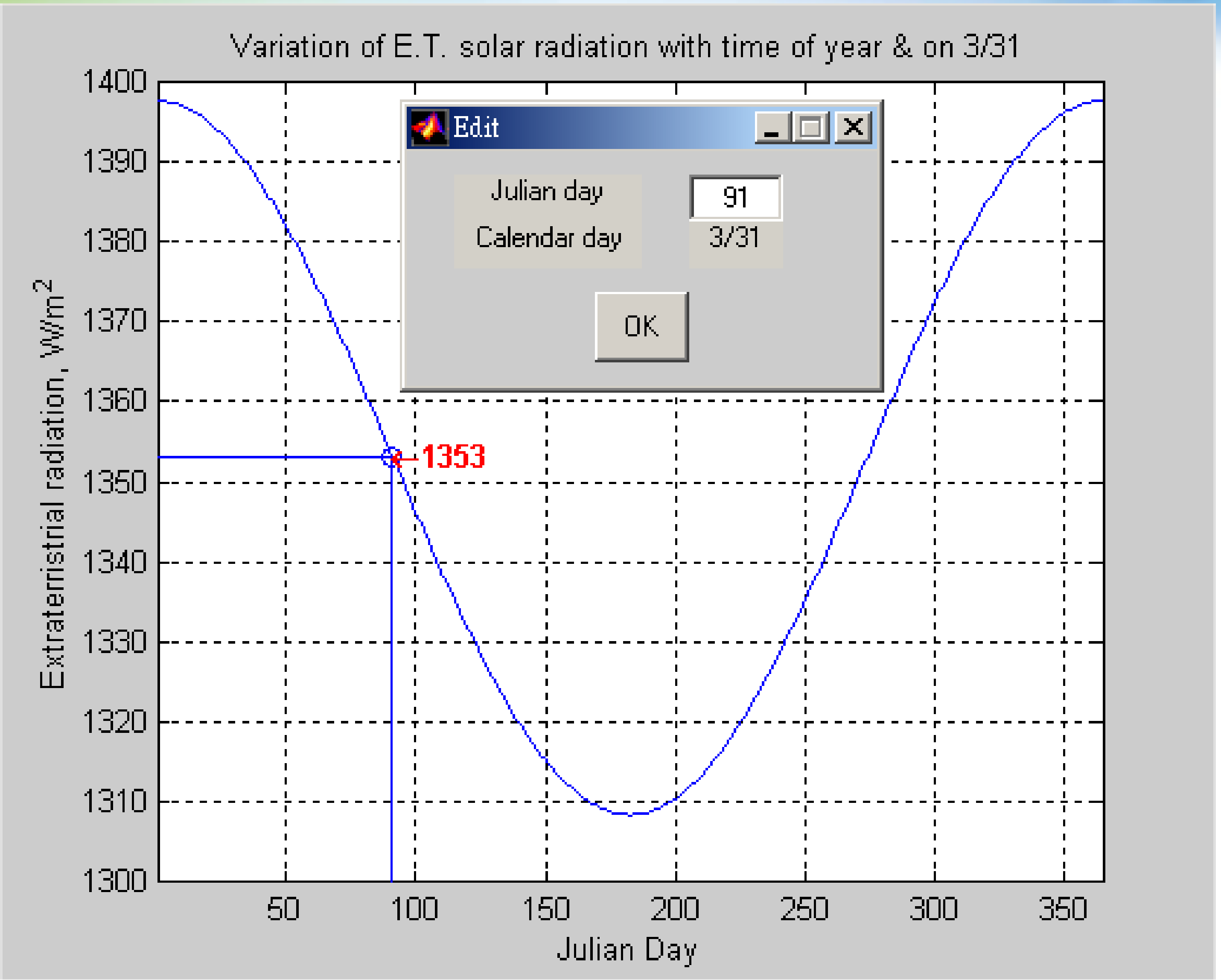


圖 5.　大氣層外圍正向太陽的平面上所接受到的太陽能，平均值稱為太陽常數

Emitted thermal radiation 放射熱輻射

Equation 3-48 適用於黑體(black body)，一般的物體以灰體(gray body)稱之，其所向外放射的輻射量(E_λ)在各波長與同溫度的完全黑體($E_{b\lambda}$)的輻射量均成一固定比例 ($\varepsilon = E_\lambda / E_{b\lambda}$)，, ε 定義為該物體的放射率(emittance)，所有E_λ之積分可用下式計算：

$$q'' = \varepsilon \sigma T^4 \text{ ……(3-49)}$$

在地表面太陽輻射能的最大值大約在0.75-0.9 kW/m^2.

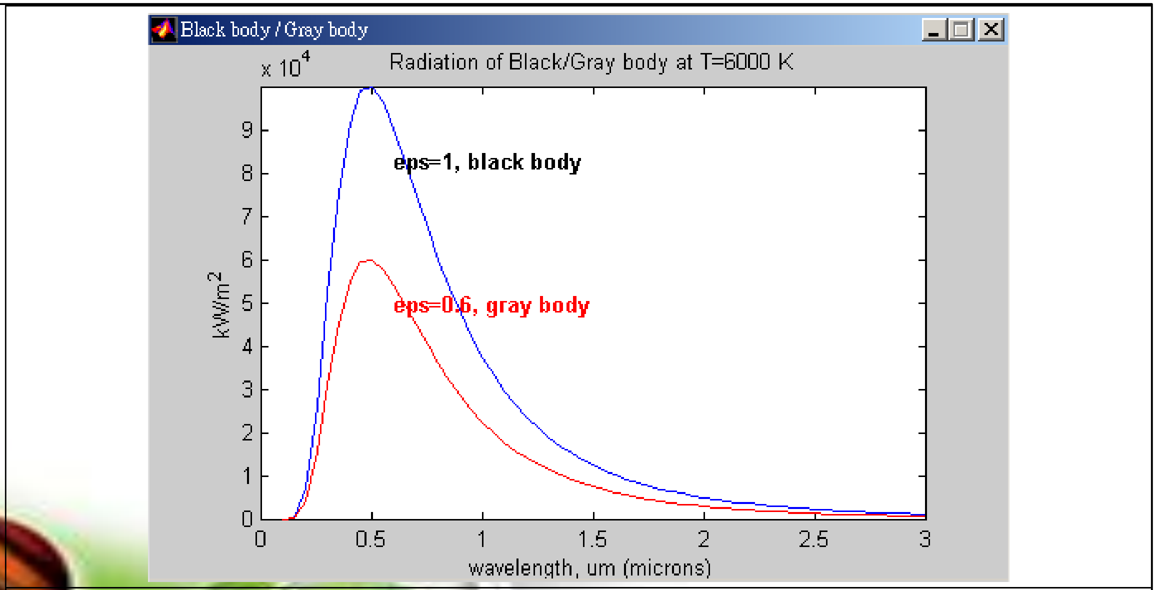


圖 6. 在 Matlab Command Window 中輸入 plancks(4) 的計算結果

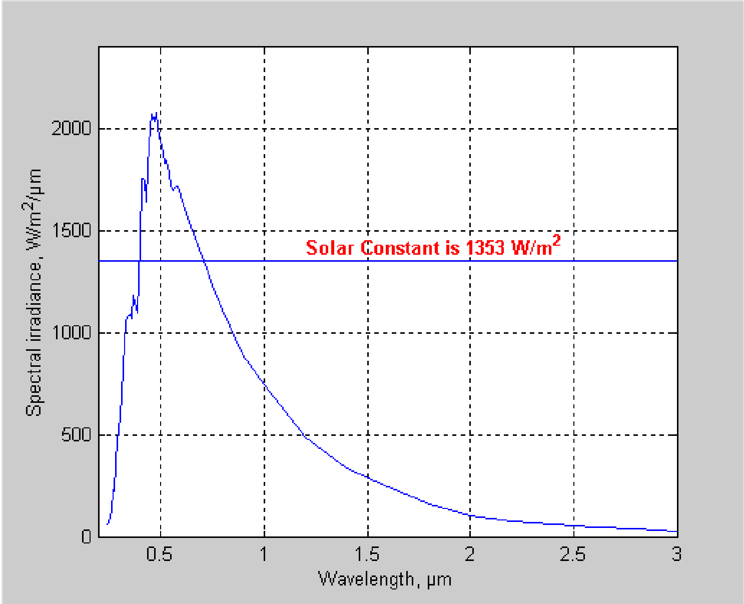


圖 7. 大氣層外圍太陽輻射能的光譜分布與太陽常數

Ex. 3-12 (p.79)

Q: A steam heating pipe has a surface T of 90 ^{o}C,

If the surface was painted with aluminized paint, $\varepsilon = 0.45$

If the surface was painted with an oil base or latex paint, $\varepsilon = 0.95$

What is the radiant flux leaving the surface?

Sol:

$$q'' = 0.45 * 5.6697 * (363.15/100)^4 = 444 \text{ W/m}^2$$

$$q'' = 0.95 * 5.6697 * (363.15/100)^4 = 937 \text{ W/m}^2$$

Difference = 937 – 444 = 493 W/m^2

Reflected & Transmitted thermal radiation 反射與穿透熱輻射

任一材料在不同波長的入射光線時有不同的放射率 (emittance)、吸收率(absorptance)、反射率 (reflectance)與穿透率 (transmittance)等光學性質。前二者彼此相等，稱為克希霍夫定律(Kirchhoff's law)，後三者相加為 1。以下證明Kirchhoff's law：

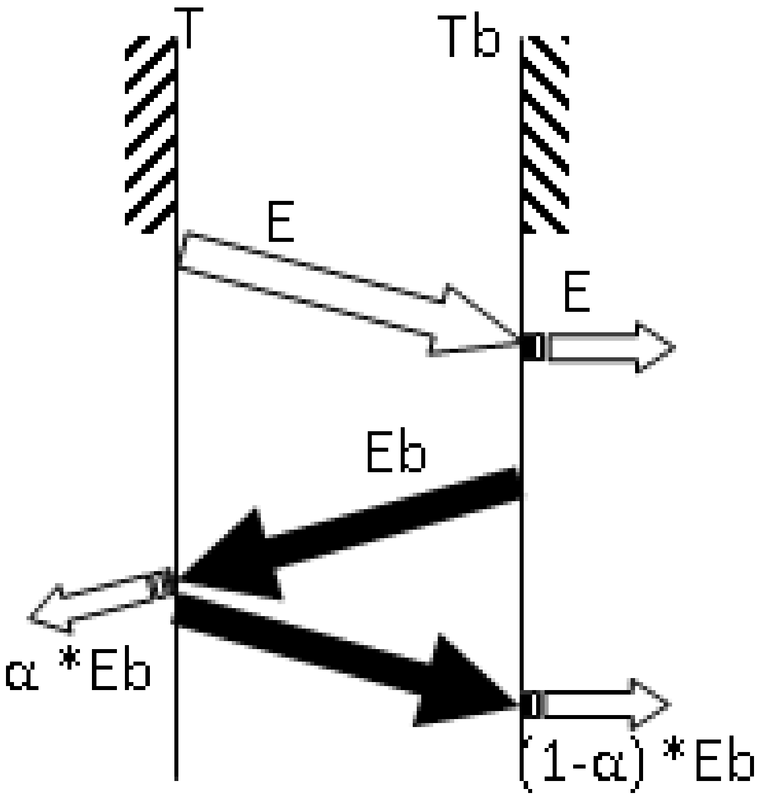
如圖所示，假設有相對的一個灰體($\varepsilon < 1$)與黑體($\varepsilon = 1$)，兩者溫度相等，(T = Tb)。由灰體與黑體輻射出去的能量分別為E與Eb，灰體的放射率 $\varepsilon = E / Eb$，吸收的能量為$\alpha * Eb$。由黑體吸收的能量為$E + (1-\alpha) * Eb$。因為兩者等溫，所以兩者的竟能量變化應該為0。

$E - \alpha * Eb = 0$ (由灰體觀之)

且 $E + (1-\alpha) * Eb - Eb = 0$ (由黑體觀之)

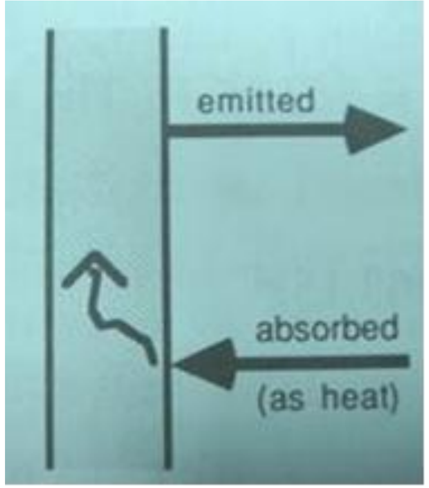
以上任一式均可導出 $E = \alpha * Eb$ → $\alpha = E / Eb$

由於 E / Eb 已被定義為等於灰體的放射率(ε)。最終得證 $\alpha = \varepsilon$



Absorptance and Emittance 吸收率與放射率

就每一波長而言，同一材料的吸收率與放射率為等值。如Appendix 3-3所示，白漆表面的放射率(ε)高達0.89-0.97，表示其吸收率(α)也有這麼高。又已知白色的反射率(ρ)是頗高的，如此則兩者再與穿透率(τ)相加必然超過 1，此結果與前述之吸收率(absorptance)、反射率(reflectance)與穿透率(transmittance)三值相加為 1之結果不符。此點可能困惑不少人。



放射率與吸收率的數值相同，然而放射率有分短波長與長波長，一般會探討吸收率的多半為長波長範圍(遠紅光)的入射熱輻射線。

前例所言，放射率高達0.89-0.97者為長波放射率，以白漆塗過的表面其放射率(吸收率)在短波範圍是很小的，如右表最末一列所示，範圍在0.12 – 0.16。高反射率(吸收率低)指的是可見光附近的短波長範圍。

表面	吸收率	
	短波輻射	長波輻射
鋁，拋光	0.15	0.04
銅，細磨光	0.18	0.03
鑄鐵	0.94	0.21
不鏽鋼，301 號 磨光	0.37	0.60
白大理石	0.46	0.95
瀝青	0.90	0.90
紅磚	0.75	0.93
礫石	0.29	0.85
平光黑漆	0.96	0.95
白漆和各種顏料	0.12-0.16	0.90-0.95

註：摘自Holman, J.P. 1976. Heat Transfer

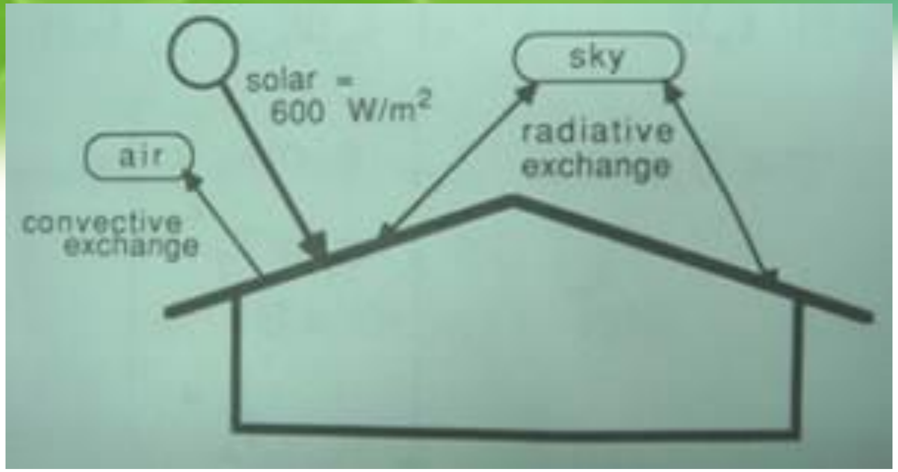
綜合性(mix mode)熱傳遞

Ex. 3-16

太陽光照射於畜舍屋頂的輻射能為 600 W/m^2. 屋頂對於太陽能的吸收率為0.6。屋頂與天空會有輻射熱交換，屋頂的輻射率為0.9，天空的溫度可用Swinbank 模式來計算。

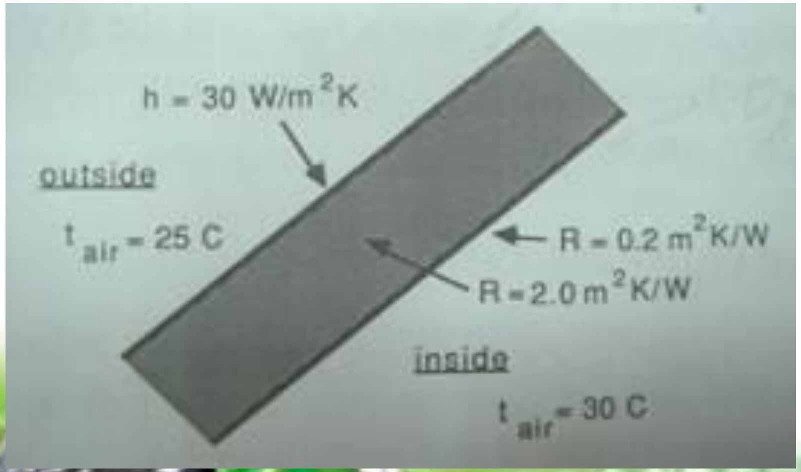
$$T_{sky} = 0.0552\ T_{air}^{1.5} \qquad \ldots\ldots\ (3\text{-}67)$$

屋頂與室外空氣也存在對流熱傳遞，對流係數為 30 W/m^{2}K, 室外溫度為25^{o}C。屋頂有絕熱保溫，熱阻為 2 m^{2}K/W. 屋頂下的室內空氣為30^{o}C，屋頂內表面與與室內空氣的熱阻為 0.2 m^{2}K/W. 此熱阻包括了對流與輻射兩種熱傳遞方式。請依據以上條件計算屋頂內表面的溫度。



$$T_{sky} = 0.0552(25\text{ C} + 273.15)^{1.5},$$
$$= 284.19\text{ K } (= 11\text{ C, or 14 K below air temperature}).$$

$$\text{gains} = \text{losses},$$
$$q''_{solar} = q''_{convective} + q''_{radiative} + q''_{conductive}.$$



Absorbed solar flux is calculated as

$$q''_{solar} = (0.60)\ (600\text{ W/m}^2) = 360\text{ W/m}^2.$$

Convective heat loss is

$$q''_{convective} = h\Delta T = 30\text{ W/m}^2\text{K}(T_{us} - 298.15\text{ K}).$$

Absolute temperatures will be used in all terms of the energy balance because they are required for radiative heat transfer calculations.

Conductive heat transfer is ($R = 2.2$ from T_{us} to the inside air)

$$q''_{conductive} = \Delta T / R = (T_{us} - 303.15\text{ K})/2.2\text{ m}^2\text{K/W}.$$

Radiation heat transfer between the barn's roof and the sky can be considered a situation of a relatively small object in large surroundings. Thus thermal radiation loss to the sky can be written

$$q''_{radiation} = \varepsilon_{us}\,\sigma\,(T_{us}^4 - 284.19^4)$$
$$= (0.9)(5.6697E{-}8)(T_{us}^4 - 65.228E + 8),$$
$$= 5.1E{-}8T_{us}^4 - 332.7.$$

$$360\text{ W/m}^2 = 30(T_{us} - 298.15)$$
$$+ (T_{us} - 303.15) / 2.2 + 5.1(T_{us}/100)^4 - 332.7$$

or $\quad 5.1(T_{us}/100)^4 + 30.4545T_{us} = 9775.3.$

Tus, K	LHS
310	9911.89
305	9729.96
306	9766.23
306.5	9784.39
306.25	9775.31

Tus = 306.25 K = 306.25-273.15

= 33.1 °C

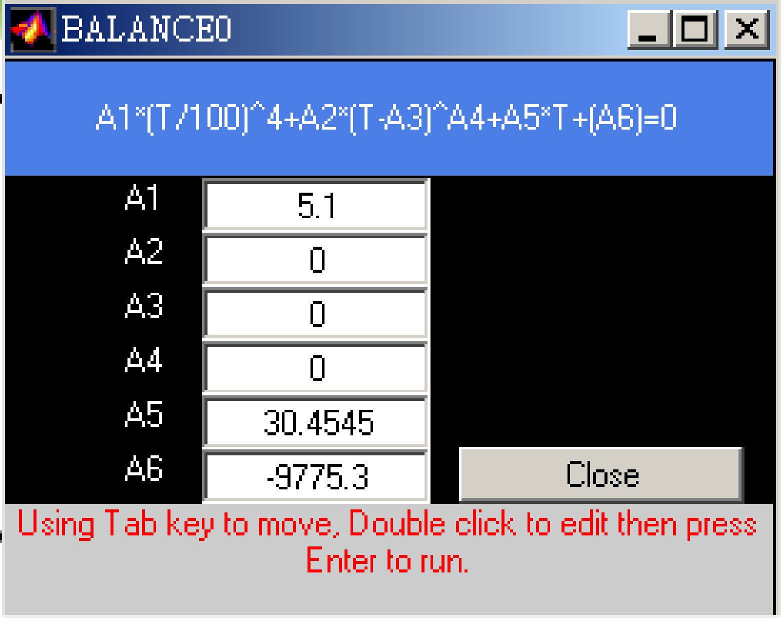


圖 11. 在 Matlab Command Window 輸入 balance0. (For Ex.3-16)

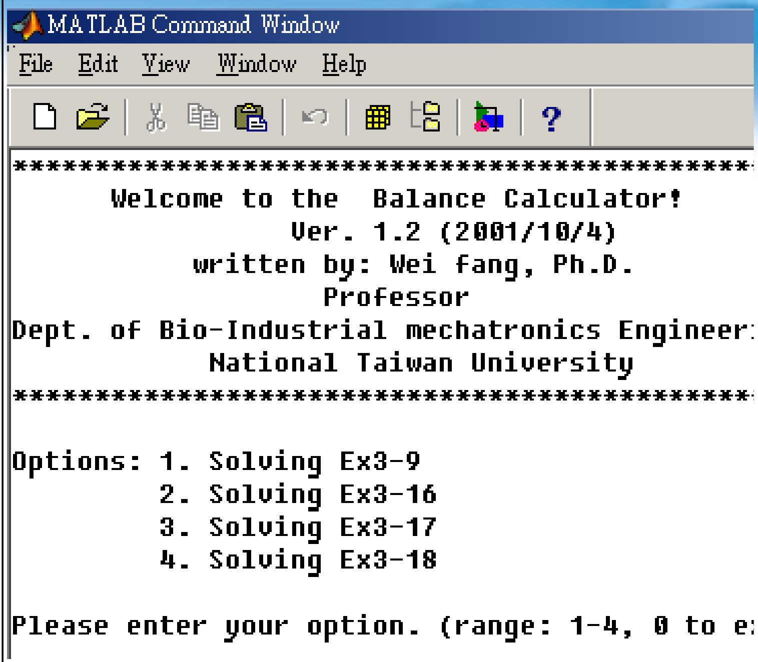
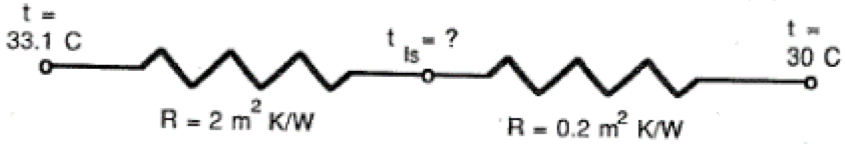


圖 12. 在 Matlab Command Window 輸 balance1. (For Ex.3-9,16-18)

以上只計算出 屋頂外表面的溫度 (33.1 度C) ，
仍需計算屋頂下表面的溫度，
此溫度決定了室內養殖的動物是否處於適當的環境內。



This is a series thermal circuit, thus, temperature differences scale linearly with resistances. The temperature of the lower surface of the roof is

$$t_{ls} = 33.1\ C + (2.0/2.2)(30\ C - 33.1\ C) = 30.3\ C.$$

APPENDIX 3-5
SURFACE CONDUCTANCES AND RESISTANCES FOR AIR
(h IN W/M^2K; R IN M^2K/W)

A. Standard data for still air, as inside a building:

Orientation of surface	Heat Flow Direction	Surface Emittance 0.90 h	0.90 R	0.20 h	0.20 R	0.05 h	0.05 R
vertical	horizontal	8.29	0.12	4.20	0.24	3.35	0.30
horizontal	upward	9.26	0.11	5.17	0.19	4.32	0.23
	downward	6.13	0.16	2.10	0.48	1.25	0.80
45 degree slope	upward	9.09	0.11	5.00	0.20	4.15	0.24
	downward	7.50	0.13	3.41	0.29	2.56	0.39

B. Moving air, as outside a building, surface in any orientation:

Wind Velocity, m/s	h	R	
6.7	34.08	0.030	(for winter)
3.4	22.72	0.044	(for summer)

NOTES: 1. Based on data in the ASHRAE Handbook of Fundamentals.

平滑的金屬管

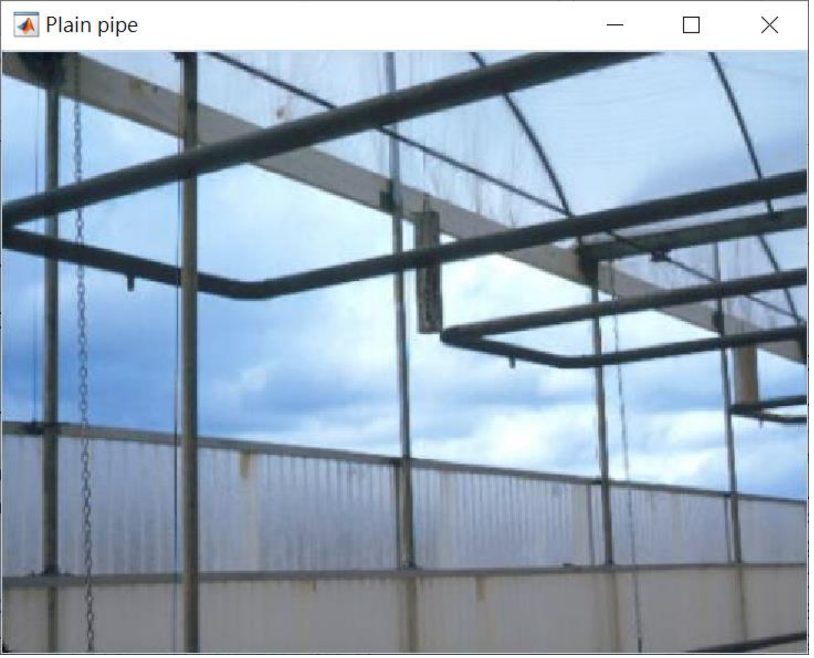


Photo depicts what is plain pipe

平滑的金屬管路
經常安裝於溫室上方或植床下方

Detail Info. Select a PLAIN Pipe out of 6 types

PHOTO DEFAULT Heat per unit length

HEAT available from various PLAIN PIPE
heated by HOT water or STEAM
from P.V. Nelson's 'Greenhouse Operation and Management'
< assuming that the inside air temperature is 60 degree F (15.56 degree C) >

HEAT SOURCE	PIPE DIAMETER		HEAT SUPPLIED (Btu/hr/ft)	(W/m)
1. RUTGERS SYSTEM	2	inch (51 mm)	170	163.45
2. Steam 215 F (102 C)	1.5	inch (38 mm)	210	201.91
3. Steam 215 F (102 C)	1.25	inch (32 mm)	180	173.06
4. Hot Water 180 F (82 C)	2	inch (51 mm)	160	153.83
5. Hot Water 203 F (95 C)	2	inch (51 mm)	200	192.29
6. YOUR design			170	163.45

平滑的金屬管路每米長度的供熱能力計算

Detail Info. Select a PLAIN Pipe out of 6 types

PHOTO DEFAULT Heat per unit length

from P.V.

< assuming that th

HEAT SOURCE

- 1. RUTGERS SYS
- 2. Steam 215 F (1
- 3. Steam 215 F (1
- 4. Hot Water 180 F
- 5. Hot Water 203 F
- 6. YOUR design

.29

.45

Heat per m of plain pipe

Steam or hot water temperature in plain pipe, °C

82

T_{air}, in deg.C

22

Mean Radiant Temp., °C

10

Surface emittance of the pipe

0.95

Outside diameter of the pipe, mm

56

Thickness of the pipe, mm

5

Thermal conductivity of the pipe (check the msgbox for reference)

60

OK Cancel

Solve T for eq. listed:

$A_1*(T/100)^4+A_2*(T-A_3)^{A_4}+A_5*T+A_6=0$, where A1 =

5.3862

A2 =

2.7135

A3 =

295.15

A4 =

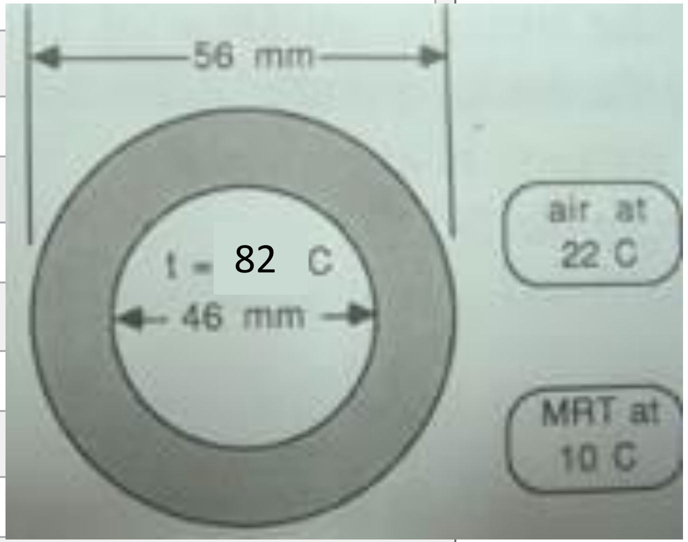
1.25

A5 =

10893.4672

A6 =

-3869161.1026



Heat supplied per m

Tsurface of the pipe is 355.06 K = 81.91 deg.C

Qcond. (962.1 W/m^2)

= Qconv. (452.3 W/m^2)

+ Qrad. (509.8 W/m^2)

Heat supplied is 169.3 W/m

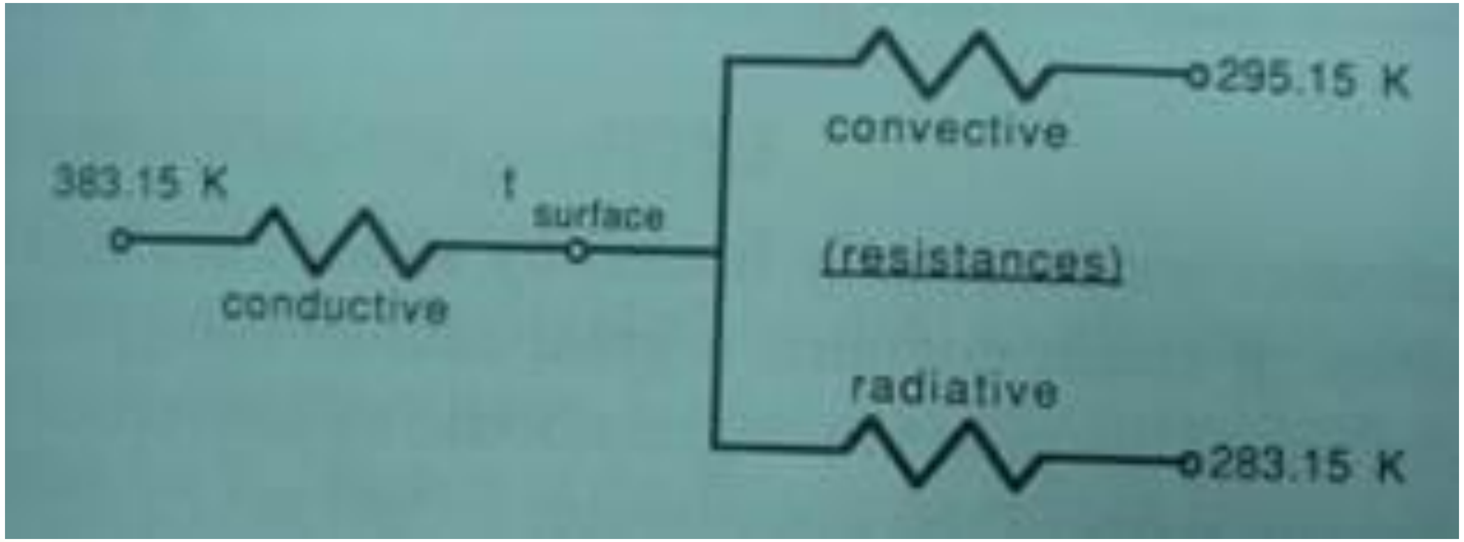
OK

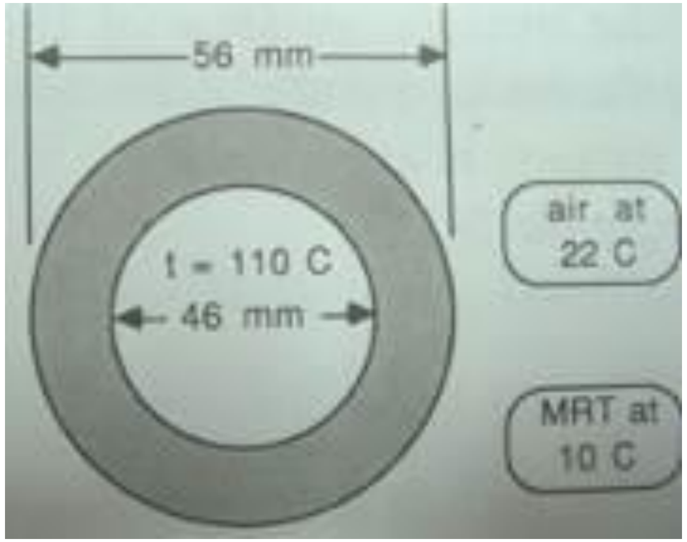
Material	Thermal Conductivity, W/mK
Metals	
Aluminum (alloy 1100)	221
Brass, red (85% Cu, 15% Zn)	150
Brass, yellow (65% Cu, 35% Zn)	120
Copper (electrolytic)	393
Gold	297
Iron, cast	47.7 (327 K)
Iron, wrought	60.4
Lead	34.8
Nickel	59.5
Silver	424
Steel, mild	45.3
Tin	64.9
Zinc, galvanizing	110

Ex. 3-17

溫室的加熱方式中有一種是使用蒸汽通入金屬管，透過金屬管外表面與室內空氣的對流與輻射兩種熱傳方式進行加熱。假設金屬管外與內徑分別為 56 mm, 46 mm，金屬管本身的熱傳導係數 (k) 為 52 W/mK. 加壓蒸汽的冷凝溫度為 110 ^{o}C，假設此亦為金屬管的內表面溫度。溫室的室內空氣為22 ^{o}C，溫室內加熱金屬管周遭的平均熱輻射溫度為 10 ^{o}C。

假設金屬管表面塗成黑色，表面的熱放射率/輻射率(ε) 為 0.95，請問金屬管與溫室之間的淨熱交換率是多少? 其中有多少是對流，有多少是輻射？





對金屬管的表面而言，傳導過來的熱 = 對流 + 輻射流失的熱

依據圓柱座標熱阻公式

$$R_{conductive} = (\ln(r_o / r_i))/2\pi kL$$
$$= (\ln(28/23))/2\pi(52)(1) \quad \text{(per unit length)}$$
$$= 0.000602 \text{ mK/W.}$$

每米長度有 0.1759 m^2 的表面積

$$R_{conductive} = (0.000602 \text{ mK/W})(0.1759 \text{ m}^2\text{/m})$$
$$= 0.000106 \text{ m}^2\text{K/W,}$$

透過熱傳導，傳到外表面的熱能計算如下：

$$q''_{conductive} = \Delta T / R$$
$$= (383.15 \text{ K} - T_{surface}) / 0.000106 \text{ m}^2\text{K/W.}$$
$$= 9441(383.15 - T_{surface})$$

加熱管輻射出的熱量假設全被溫室內空間吸收，透過輻射的熱損失計算如下：

$$q''_{radiative} = (0.95)(5.6697E-8)(T^4_{surface} - 283.15^4)$$
$$= 5.386(T_{surface}/100)^4 - 346.3$$

由已知並無法確認管路外的對流是屬於層流或是紊流。但可透過 Eq 3-34 的分析，計算如果是紊流，所需要的溫差 (△)T

$L^3 \Delta T = 1; L = 0.056$ m. $\Delta T = 1 / L^3 = 5694$ K.

以溫室環境來看溫差不可能那麼大，所以一定是層流，所以可選表 3-2 (p.65) 中公式 3-40 來計算對劉熱傳係數 (h)

$$h = 1.32\,((T_{surface} - 295.15\text{ K}) / 0.056\text{ m})^{0.25},$$
$$= 2.713\,(T_{surface} - 295.15)^{0.25}.$$

由加熱管表面對溫室的對流熱傳遞計算如下：

$$q''_{convective} = h\Delta T = 2.713(T_{surface} - 295.15)^{1.25}.$$

Conductive gain = Radiative loss + Convective loss

$$9441(383.15 - T_s) = 5.386(T_s/100)^4 - 346.3 + 2.713(T_s - 295.15)^{1.25}$$

$$5.386*(Ts/100)^4+2.713*(Ts-295.15)^{1.25}+9441*Ts-3617760 = 0$$

求解得出 Ts = 382.9967 K，代回上式，求出以下各項：

傳導： 1542 W/m^2 單位長度傳導的熱量為 1542 * 0.1759 m^2/m = 271.2 W/m

對流： 812 W/m^2 輻射： 730 W/m^2

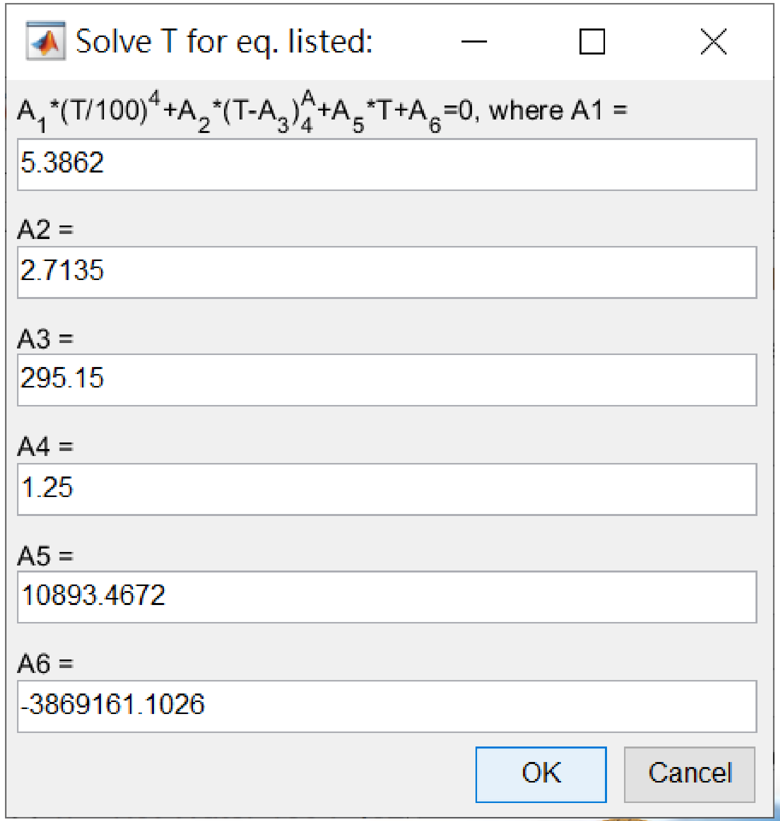
Mixed mode 的通式

$$A1*(Ts/100)^4 + A2*(Ts-A3)^{A4} + A5*Ts + A6 = 0$$ ...[3-69]

傳導 Conduction related parameters: A5, A6
對流 Convection related parameters: A2, A3, A4
輻射 Radiation related parameters: A1, A6

前兩例只有流體溫度與熱傳導係數不同，只影響熱傳導。
換言之，應該只有 A5, A6 有不同數值。

$$5.386*(Ts/100)^4+2.713*(Ts-295.15)^{1.25}+9441*Ts-3617760 = 0$$



Finned pipe

Photo depicts what is finned pipe

多鰭片的金屬管路
經常安裝於溫室靠外側的牆面

Detail Info. Select a FINNED Pipe out of 6 types

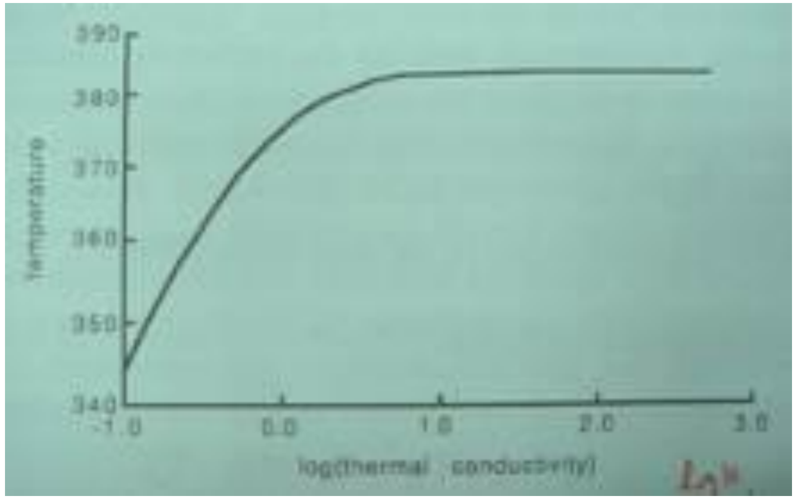
PHOTO DEFAULT

HEAT available from various FINNED PIPE
heated by HOT water or STEAM
< assuming that the inside air temperature is 60 degree F (15.56 degree C) >

HEAT SOURCE	PIPE DIAMETER	HEAT SUPPLIED (Btu/hr/ft)	(W/m)
1. RUTGERS SYSTEM	2 inch (51 mm)	950	913.39
2. Steam 215 F (102 C)	1.75 inch (44 mm)	1800	1730.64
3. Steam 215 F (102 C)	1.25 inch (32 mm)	1200	1153.76
4. Hot Water 180 F (82 C)	2 inch (51 mm)	950	913.39
5. Hot Water 203 F (95 C)	2 inch (51 mm)	950	913.39
6. YOUR design		950	913.39

Ex. 3-18 重做上例

繪圖顯示管路材料的熱傳導係數對管路外表面溫度的影響。
The thermal conductivity of the pipe wall influences terms A5 and A6 of equation 3-69.



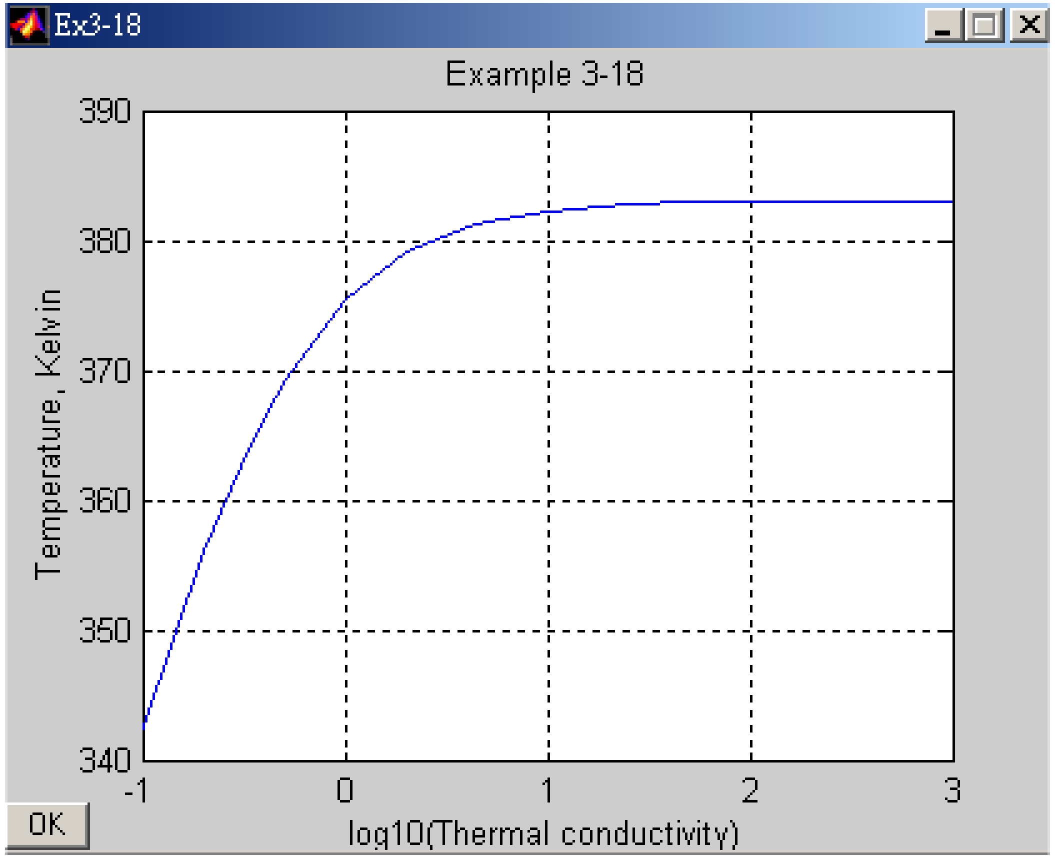
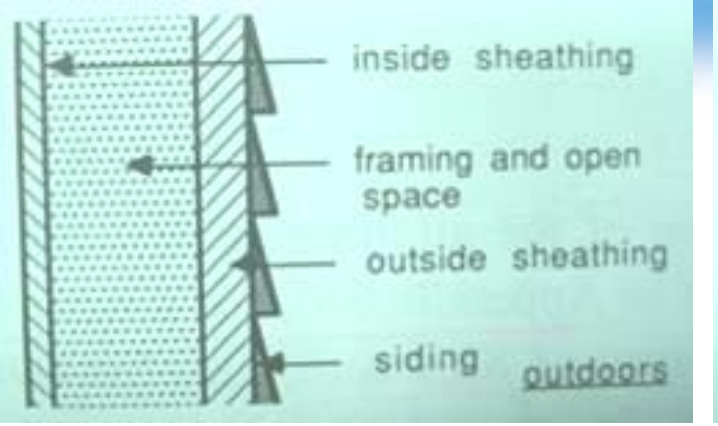


圖 13. 在 Matlab Command Window 輸入 Balance1(‘4’) 的輸出結果

平面間空隙的熱阻

effective cavity emittance 空隙的有效輻射率

$$\mathbf{E} = \left(\varepsilon_1^{-1} + \varepsilon_2^{-1} - 1\right)^{-1}$$



Ex. 3-19 (p.100) 估算牆壁中空部分的熱阻，假設中空部分的寬度為90 mm，室內外溫度分別為20 度C與 -10 度C。

假設經過空隙部分的溫差為 10 K
假設內表面的熱輻射率為 0.9
由 上式可求得effective cavity emittance ：

$$E = [\,(1/0.9) + (1/0.9) - 1\,]^{-1} = 0.82$$

由於是牆面，所以
position of air space 是 vertical
Direction of heat flow 是 horizontal

由附錄 3-6 (p.398) 可查得
當空隙平均溫度 Mean airspace T = 10 K
空隙厚度 airspace thickness 88.9 mm

dT = 16.7 K → thermal resistance of 0.16 m^2K/W
dT = 5.6 K → thermal resistance of 0.18 m^2K/W

在本案例 dT = 10 K, 內插可求得
空隙的thermal resistance 熱阻 = 0.17 m^2K/W.

APPENDIX 3-6

THERMAL RESISTANCES OF PLANE AIR SPACES

Taken from the 1985 ASHRAE Handbook of Fundamentals, American Society of Heating, Refrigerating and Air Conditioning Engineers, Atlanta GA

Table 2A Thermal Resistance of Plane Air Space

$m^2.{}^oC/W$

已知

- position of air space 是 vertical
- Direction of heat flow 是 horizontal
- Mean airspace T = 10 K, airspace thickness 88.9 mm,
- effective cavity emittance (E) of 0.82,
- 經過空隙部分的溫差為 10 K

空隙的溫差 / 空隙的厚度 / 空隙的走向 / 熱流的方向

Position of Air Space (空隙的走向)	Direction of Heat Flow (熱流的方向)	Air Space (空隙的溫差): Mean Temp.[d] (°C)	Air Space (空隙的溫差): Temp Diff.[d] (°C)	38.1 mm Al (空隙的厚度): Value of 0.03	0.05	0.2
Horiz	Up ↑	32.2	5.6	0.45	0.42	0.3
		10.0	16.7	0.33	0.32	0.2
		10.0	5.6	0.44	0.42	0.3
		-17.8	11.1	0.35	0.34	0.2
		-17.8	5.6	0.43	0.41	0.3
		-45.6	11.1	0.34	0.34	0.3
		-45.6	5.6	0.42	0.41	0.3
45° Slope	Up ↗	32.2	5.6	0.51	0.48	0.3
		10.0	16.7	0.38	0.36	0.2
		10.0	5.6	0.51	0.48	0.3
		-17.8	11.1	0.40	0.39	0.3
		-17.8	5.6	0.49	0.47	0.3
		-45.6	11.1	0.39	0.38	0.3
		-45.6	5.6	0.48	0.46	0.3
Vertical	Horiz. →	32.2	5.6	0.70	0.64	0.4
		10.0	16.7	0.45	0.43	0.3
		10.0	5.6	0.67	0.62	0.4
		-17.8	11.1	0.49	0.47	0.3
		-17.8	5.6	0.62	0.59	0.4
		-45.6	11.1	0.46	0.45	0.3
		-45.6	5.6	0.58	0.56	0.4

空間厚度

effective cavity emittance

38.1 mm Air Space[c]					88.9 mm Air Space[c]			
Value of $E^{d,e}$					Value of $E^{d,e}$			
0.05	0.2	0.5	0.82	0.03	0.05	0.2	0.5	0.82
0.42	0.30	0.19	0.14	0.50	0.47	0.32	0.20	0.14
0.32	0.26	0.18	0.14	0.27	0.35	0.28	0.19	0.15
0.42	0.32	0.21	0.16	0.49	0.47	0.34	0.23	0.16
0.34	0.29	0.22	0.17	0.40	0.38	0.32	0.23	0.18
0.41	0.33	0.24	0.19	0.48	0.46	0.36	0.26	0.20
0.34	0.30	0.24	0.20	0.39	0.38	0.33	0.26	0.21
0.41	0.35	0.27	0.22	0.47	0.45	0.38	0.29	0.23
0.48	0.33	0.20	0.14	0.56	0.52	0.35	0.21	0.14
0.36	0.28	0.20	0.15	0.40	0.38	0.29	0.20	0.15
0.48	0.35	0.23	0.17	0.55	0.52	0.37	0.24	0.17
0.39	0.32	0.24	0.18	0.43	0.41	0.33	0.24	0.19
0.47	0.37	0.26	0.20	0.52	0.51	0.39	0.27	0.20
0.38	0.33	0.26	0.21	0.41	0.40	0.35	0.27	0.22
0.46	0.39	0.30	0.24	0.51	0.49	0.41	0.31	0.24
0.64	0.40	0.22	0.15	0.65	0.60	0.38	0.22	0.15
0.43	0.32	0.22	0.16	0.47	0.45	0.33	0.22	0.16
0.62	0.42	0.26	0.18	0.64	0.60	0.41	0.25	0.18
0.47	0.37	0.26	0.20	0.51	0.49	0.38	0.27	0.20
0.59	0.44	0.29	0.22	0.61	0.59	0.44	0.29	0.22
0.45	0.38	0.29	0.23	0.50	0.48	0.40	0.30	0.24
0.56	0.46	0.34	0.26	0.60	0.58	0.47	0.34	0.26